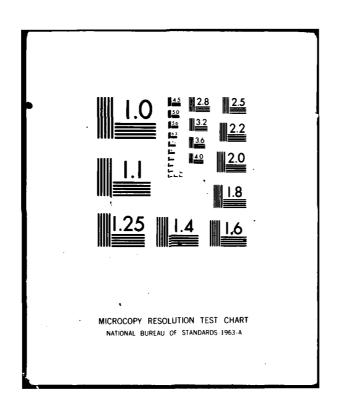
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# NAVAL POSTGRADUATE SCHOOL Monterey, California





Masters THESIS

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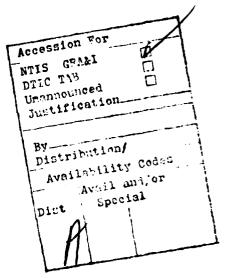
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with respect to common trends and differences in the behavior as well as with respect to the usefulness of the Bayes' estimation technique underlying the model.

It is found that the model yields reliable estimates of the retention rates that can provide a meaningful substitute for actually observed rates especially within prediction models. The author finally recommends an approach that extends his model into a prediction model for the retention-rates of P-coded naval officers.



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Estimation of Retention Rates of P-Coded Naval Officers
Conditioned on Their
Year of Graduation and Subspecialty Code

by

Heinz Dieter Mueller Captain, German Air Force Betriebswirt (grad.), Fachhochschule des Heeres 1, 1974

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### **ABSTRACT**

This thesis analyzes retention behavior of P-coded naval officers and derives a model to estimate retention rates for this group of officers conditioned on their respective year of graduation and the Subspecialty-code obtained following completion of postgraduate education. The model is based on Bayes' estimation technique.

The estimates resulting from applying the model to the data obtained by observing the behavior of 3,981 naval officers who were graduated between 1970 and 1975 and who obtained one of 41 selected Subspecialty-codes are analyzed with respect to common trends and differences in the behavior as well as with respect to the usefulness of the Bayes' estimation technique underlying the model.

It is found that the model yields reliable estimates of the retention rates that can provide a meaningful substitute for actually observed rates especially within prediction models. The author finally recommends an approach that extends his model into a prediction model for the retention-rates of P-coded naval officers.

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### I. INTRODUCTION

The estimation of retention rates for P-coded naval officers has been a major problem for personnel planners. The procedure currently used by the Navy overestimates the actual need. 1

One of the reasons for this overestimation is the fact that the samples available are too small to provide a basis for reliable estimation by traditional procedures.

In a Memorandum of 21 September 1979, Weitzman<sup>2</sup> suggests two approaches to a solution to this problem:

- a. Aggregation of subsamples to increase the reliability of traditional estimators;
- b. Development of new estimation models.

One of the basic devices to cope with the problem of too small sample sizes in this context is the Bayesian estimation technique. It could provide the basis for a new estimation model.

It is the purpose of this study to derive a Bayesian estimator which will yield reliable estimates of retention rates of P-coded naval officers conditioned on their respective year of graduation and their respective Subspeciality-code obtained after graduation.

Weitzman, R.A., Memorandum ND4(54Wz)/bd 21 Sept 1979: Naval Officer Subspecialty Analysis.

<sup>&</sup>lt;sup>2</sup>See Weitzman, R. A., Memorandum NC4

The data base for this study consisted of 3,981 naval officers who were graduated in the years 1970 to 1975 and who attained one of 41 selected Subspeciality-codes which are related to the different curricula of the Naval Postgraduate School. The retention behavior of these officers was observed from the year of graduation up to the year 1979, which was the last year that corresponding data were obtained from the Officer Master File and Attrition File. The observed retention rates are analyzed mainly with respect to

- differences in the behavior of the graduates over the six subsequent graduation years,
- trends within the respective graduation-year groups over years k after graduation, and
- differences between the ten Subspecialty-code groups into which the 41 Subspecialty-codes were grouped.

Based on the Bayesian estimator for this study and the specific phenomena observed, a model is formulated that allows use of the estimator to obtain an estimate of the retention rate that can provide a more reliable substitute than the actually observed rate, especially within prediction models.

The estimates are calculated and their usefulness as a reliable basis for prediction models is analyzed. The analysis and discussion of the obtained results are concluded by correlating the estimates with their respective rates. This procedure led to final conclusions about the actual retention behavior as well as about the usefulness of the Bayesian esimation technique within this context.

Finally, recommendations are given with respect to possible extensions of the derived model into prediction models.

# II. BAYESIAN ANALYSIS

## A. IDEA OF BAYESIAN ANALYSIS

Bayesian Analysis is an attempt to incorporate into the process of statistical inference all information about the underlying state of nature of a random phenomenon. It tolerates explicitly the use of subjective judgement where a priori verifiable information is not available. 3

Suppose there exists a set of mutually exclusive events, say officer A remains in service or he leaves service in a certain year k and a priori there exists no certainty about his behavior. Then the Bayesian estimation technique allows for assigning prior probabilities to each of these events on the basis of whatever evidence is known or subjectively assumed in advance. Then, if additional facts become available, for example retention rates for a group of officers to whom officer A may belong, the initial probabilities are revised by means of Bayes' Theorem. As a result of this revision process, posterior probabilities are obtained. They do not completely supersede the prior information, but they contain it.

Whether this prior information is still useful after additional evidence has been obtained depends on the phenomenon under consideration. In theory as well as in practice

Morgan, B. W., An Introduction to Bayesian Statistical Decision Processes, p. 1-14, and Preface, Prentice-Hall, 1968.

the above described distinctive feature of Bayesian analysis is the subject of controversial opinions. Examples of bizarre results of Bayesian analysis can be found in almost all standard statistics books.

# B. BASIC STRUCTURE OF BAYESIAN ANALYSIS<sup>5</sup>

Suppose that it is possible to summarize a priori information about the relative likelihood of where in a specified parameter space  $\Omega$  the unknown value of parameter  $\theta$  lies by constructing a probability distribution for  $\theta$  on  $\Omega$ .

Assuming a continuous case, the p.d.f.  $\xi(\theta)$  of this distribution is then called the prior p.d.f. of  $\theta$ .

Suppose then that a random sample  $X_1, \ldots, X_n$  (Vector notation: X) is drawn from a distribution with p.d.f.  $f(x|\Theta)$ , where the value of  $\Theta$  is unknown and the prior distribution of  $\Theta$  has the p.d.f. $\xi(\Theta)$ . The joint p.d.f. of the random variables  $X_1, \ldots, X_n$  is in vector notation  $f(X|\Theta)$  with marginal joint p.d.f.

(1)  $f(x) = \int_{\Omega} f(x|\theta) \, \xi(\theta) \, d\theta$ . Then after  $x_1 = x_1, \ldots, x_n = x_n$  has been observed, the conditional p.d.f. of  $\theta$  given these values

Example in Wonnacott, T. H., and Wonnacott, R. J., Introductory Statistics, Third Edition, p. 591-593, John Wiley Sons, 1977.

The argumentation in this chapter is based on:
DeGroot, M. H., Probability and Statistics, Menlo Park, 1975.
Winkler, R. L., Introduction to Bayesian Inference and Decision,
Holt, Rinehart and Winston, Inc., 1972. Wonnacott, Introductory
Statistics, 1977.

(2) 
$$\xi(\Theta | \tilde{X}) = \frac{\tilde{f}(\tilde{X} | \Theta) \xi(\Theta)}{\int_{\Omega} \tilde{f}(\tilde{X} | \Theta) \xi(\Theta) d\Theta}$$
 for  $\Theta$  in  $\Omega$ 

is denoted as the posterior p.d.f., where  $f(X|\Theta)$  represents the likelihood function and  $\xi(\Theta)$  the prior distribution.

Equation (2) is exactly the Bayes Theorem for continuous random variables. As Winkler<sup>6</sup> states, this equation provides conceptually a convenient way to revise prior information when additional evidence by means of sample information is obtained. However, except for relatively simple mathematical functions, it might prove to be impossible to carry out the required integration.

These potential difficulties led to resorting to the concept of conjugate prior distributions: in essenence families of distributions for which the likelihood function  $\xi(\Theta|X)$  is uniquely determined once a data-generating model is specified.

Conjugate families of distributions corresponding to some likelihood functions that are important for practical purposes have been developed. In the context of this study, in which sampling from a stationary and independent Bernoulli process is the underlying data-generating model, the conjugate family is the family of Beta-distributions.

Suppose the retention behavior of a randomly chosen group of n officers has been observed.

<sup>6</sup> Winkler, R. L., p. 147.

Let  $X_1 = \begin{cases} 1 & \text{if officer 1 remained in service in year k} \\ 0 & \text{otherwise} \end{cases}$ 

Then  $X_1$ , ...,  $X_n$  form random sample from a Bernoulli distribution for which the value of  $\Theta(0 \le 0 \le 1)$  is unknown. Assuming that the prior distribution of  $\Theta$  is a Beta-distribution with parameters  $\alpha$  and  $\beta$  ( $\alpha$  > 0,  $\beta$  > 0), then the posterior distribution of  $\Theta$  given that  $X_1 = X_1, \ldots, X_n = X_n$  is a Beta-distribution with parameters

$$\alpha + \sum_{i=1}^{n} x_{i}$$
 and  $\beta + n - \sum_{i=1}^{n} x_{i}$ .

## C. DEFINITION OF THE BAYES' ESTIMATOR FOR THIS STUDY

# 1. Bayes' Estimator

Based on the observed values of the random vector X, the value of  $\theta$  can be estimated. Thus the estimator of  $\theta$  is a real valued function of  $\widetilde{X}$  denoted by  $\delta\left(X\right)$ .

To determine the goodness of the estimator in terms of the closeness of the estimate to the true value of the parameter 0, the quadratic loss function is used as suggested by most statisticians: 7

(3)  $L(\theta,a) = (\theta-a)^2$ . Hence the estimate should be chosen such that  $E[(\theta-a)^2|\tilde{X}]$  is minimal, where the expected loss is

<sup>7</sup>See DeGroot, p. 276, and Wonnacott, p. 573.

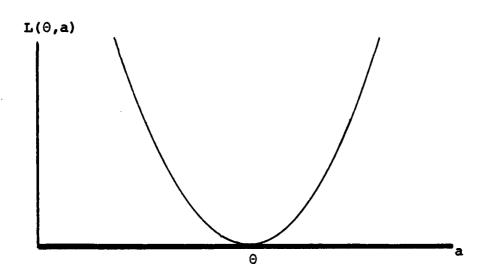


Figure I: Quadratic Loss Function for a Given  $\theta$ 

(4)  $E[(\Theta - a)^2 | \tilde{x}] = \int_{\Omega} (\Theta - a)^2 \xi(\Theta | \tilde{x}) d\Theta$ ,

and  $\xi(\theta|X)$  denotes the posterior p.d.f., defined with equation (2).

The Bayes' estimator of  $\theta$  is now the function  $\delta^*(X)$  which yields for every possible value  $\tilde{X}$  of the random vector  $\tilde{X}$  an estimate for which the loss according to equation (4) is minimal. For the Conjugate family of Betadistributions

distributions
$$(5) \delta(x) = \frac{\alpha + \sum_{i=1}^{n} x_i}{\alpha + \beta + n}, i.e.$$

 $\delta^*(X)$  is the mean of the posterior Beta-distribution. Given that the optimal estimate  $\delta^*(X)$  is found, then it is easily possible to use this revised information as new prior information for a second application of Bayes' Theorem.  $^8$ 

In this case the posterior distribution after the first application of Bayes' Theorem becomes the prior distribution for the next stage and can again be revised by newly obtained information on the basis of a second sample.

The nature of Bayesian estimation implies that the obtained optimal estimate is a compromise between two or more sets of sample information. In case of considerable differences in the sizes of obtainable samples, larger samples

<sup>8</sup>see Winkler, p. 160-162.

<sup>9</sup>Wonnacott, p. 546.

gain more weight than do smaller samples. This might lead to the effect that, independent of the quality of a set of sample information, the quantity in terms of the size of the sample used in the revision process determines the value of the estimate.

However, as long as with decreasing sample size the quality of the obtainable information decreases, this effect is desirable. Looking at P-coded officers who belong to a certain subspecialty code, it might for example be that within a certain period the behavior of only three officers is observable. If two of those officers happened to die in a car accident, the retention rate for this year, conditioned on the Subspecialty-code under consideration, would be 1/3. This mathematically "true" rate would undoubtedly be completely meaningless as an entry in a prediction model for retention behavior.

# 2. The Special Problem of a Diffuse Prior Distribution

Consider a situation in which a prior distribution has to be assessed without sufficient a priori information relative to some "overwhelming" sample information. Winkler denotes this state as being diffuse relative to the sample information. 10

Generally this diffuse state is most appropriately represented by a diffuse conjugate prior distribution,

<sup>10</sup> Winkler, p. 198.

specifically the Uniform (0,1)-distribution which is a Beta-distribution with parameters  $\alpha=1$  and  $\beta=1.$  Thus equal probabilities are assigned to all possible values of  $\Theta$  within  $\Omega$ , and according to Morgan the maximum possible error will be minimized. However, there exist situations in which the applicability of the Uniform (0,1)-distribution as a diffuse prior distribution is doubtful. 13

Suppose that, of a second sample of three officers, one remained in service in year k, i.e. the observed actual retention rate for year k is 1/3. Suppose further that prior information is represented by the Uniform (0,1)-distribution. Then the Bayes' estimate will be

$$\delta^* = \frac{1 + 1}{1 + 1 + 3} \quad \frac{2}{5} \quad .$$

This shows that in cases where extremely small samples are involved the Uniform (0,1)-distribution which was supposed to have no influence on the posterior distribution does not act as a diffuse prior distribution. For practical purposes in an inferential or decision-making situation, Winkler suggests therefore to use as parameters of the diffuse prior distribution  $\alpha=0$  and  $\beta=0.14$  From a theoretical standpoint

<sup>11</sup> See Winkler, p. 198-201 and Morgan, p. 46.

<sup>12</sup> Morgan, p. 47.

<sup>13</sup>see Winkler, p. 198-204.

<sup>14</sup> Winkler, p. 202.

this implies the involvement of an improper Beta-distribution where the total area under the p.d.f. does not equal to one. However, looking at the parameters of the prior distribution as equivalent to an a priori information status, this approach seems to be the only choice, as Winkler states it. 15

The effect of this approach on the value of the estimate is the same as the effect of a large sample size in the revision process relative to a small prior base. In both cases the posterior distribution will solely depend on the sample information.

# 3. The Bayes' Estimator Used in This Study

To estimate retention rates of officers conditioned on their respective Subspecialty codes the idea of the successive application of Bayes' Theorem will be used.

It is assumed that a priori the individual decision of an officer to remain in service in year k after he was graduated and thus obtained his P-code is appropriately represented by a diffuse prior Beta-distribution.

As the first sample information in the revision process of this prior information, the retention behavior of the group m of officers will be measured who belong to the same Subspecialty-code group. 16 The resulting posterior distribution will then be used as the prior distribution

<sup>15</sup>Winkler, p. 202

<sup>&</sup>lt;sup>16</sup>A list of the observed Subspecialty-code groups and the Subspecialty codes belonging to them is contained in Table 1 and Appendix A-1.

for one more application of Bayes' Theorem. As the second sample information, the retention behavior of a subgrouping n of officers belonging to the same Subspecialty-code within the above used Subspecialty-code group will be measured and used to revise the prior distribution obtained in the first step.

In addition to the assumption stated above, Winkler's suggestion concerning the diffuse prior distribution will be followed, i.e., in calculating the final estimate, the parameters of the original prior distribution will be treated as if they were zero.

Let

- y denote the number of officers belonging to n who left service up to and including year k after graduation and let
- z denote the number of officers belonging to m who left service up to and including year k after graduation.

Then as Bayes' estimate of the rate of officers who left service up to and including year k after graduation and belong to both of the above described groupings n and m

(6) 
$$\delta^* = \frac{2 + y}{m + n}$$
 will be used. 17

<sup>17</sup> This is one of the estimators that Professor R. A. Weitzman (NPS) has suggested for use in pattern analysis to cope with the problem of empty or near-empty cells (personal communication).

# III. DATA BASE

### A. DESCRIPTION OF THE DATA

Data for this study have been provided by the Department of the Navy. 18 They were extracted from both the Officer Master File (OMF) and the Attrition File (AF). In order to conduct the study with a data base as broad as possible, records of a total of 6,372 American Navy officers were established. The officers were selected according to three criteria:

- a. They have been assigned a "P"-code. 19
- b. They were graduated from a graduate-level program between 1960 and 1975, inclusive.
- c. They have obtained one of the 41 Subspecialty-codes (SSC) listed in Table 1.20

To be able to extract retention rates conditioned on the SSC's selected, the records obtained from above mentioned files have to represent the complete group of naval officers satisfying the three criteria. First preliminary summary statistics about the losses between 1960 and 1969 revealed that the number of lost officers seemed to be zero. It was

<sup>18</sup> Department of the Navy - NMPC, 8 April 1980.

<sup>19</sup> Suffix "P" means "Master level"; in the context of this study a P-coded officer has reached at least the Master level.

The SSC's are related to curricula at NPS (OPNAVNOTE 1520, 25 June 79).

SSC group	Subspecialty codes (SSC)										
xx2x	xx21	жж22	xx23	xx24	<b>xx</b> 25	<b>xx</b> 26	<b>x</b> x27				
жж3ж	xx31	<b>xx32</b>	жж33	<b>x</b> x34	<b>x</b> x38						
xx4x	<b>xx42</b>	<b>x</b> x44	xx48	xx49							
хх5х	xx51	<b>xx</b> 52	<b>x</b> x54	<b>x</b> x55	<b>x</b> x56						
хх6х	xx61	<b>x</b> x62	<b>x</b> x63	<b>x</b> x67							
xx7x	xx71	ж <b>ж</b> 72									
xx8x	xx81	xx82									
xx9x	xx91	<b>x</b> x95									
llxx	1101	1102	1103								
13xx	1301	1302	1304	1305	1306	1307	1308				

Table 1: List of the 41 SSC's Selected for This Study

found that the data available for this period in the form needed contain only officers who were still in service past 1969. Therefore the final data base consists of those officers fulfilling criteria a. and c. who were graduated in the years 1970 to 1975. Their total numbers for the graduation years indicated are shown at the top of the next page.

Data obtainable for each individual and sufficient to conduct the study were structured in records as shown in Appendix A-2. Table 2 contains a listing of the data. In order to be able to calculate the retention rates for each year k after graduation, ten entries for the vector X--which was

Graduation year	Total number of officers
1970	562
1971	715
1972	768
1973	669
1974	649
1975	618
Total data base	3,981

introduced in Chapter II--were added. They contain for the 10 calendar years under consideration (1970 - 1979) the value

These modified records served as the basis for any further calculations. A detailed listing of the number of officers who were graduated in the years 1970 to 1975, categorized according to their respective Subspecialty-code (SSC) is contained in Appendix B.

B. RESTRICTIONS AND ASSUMPTIONS IMPOSED BY THE NATURE OF THE DATA

The nature of the data imposed two major restrictions on the conduct of the study:

a. The OMF contains the date of graduation only in the

Entry-No.	Content
(1)	Social Security Number
(2)	Month and year of birth
(3)	Month and year of entry in the system
(4)	Year of graduation from a graduate-level program
(5)	Minimum Service Requirement (month and year)
(6)	Month and year of loss
(7)	First year eligible to retire (year)
(8)	Subspecialty-code (SSC)
	Content of an Individual Record as Obtained From Department of the Navy - NMPC

year in which this event occurred. Thus the basic time unit for this study is the calendar year.

b. The Minimum Service Requirement for each officer as obtainable from the OMF is not kept updated. Thus no analysis is possible relating the loss of a person to that date.

The loss date as extractable from OMF/AF and as contained in the records under entry-No. six 21 shows

- the actual loss date when an individual was lost prior to 1980.
- no entry or an expected future loss date when an individual was not lost prior to 1980.

As "first year eligible to retire" (RY), the OMF/AF and the records structured for this study show the calculated first possible retirement year based on the 20-year limit. This date is kept on the OMF/AF even if an individual served longer.

With regard to the loss date, three assumptions had to be made:

- a. A person lost in the year recorded as RY was lost due to reaching his retirement age.
- b. A person lost after the year recorded as RY was lost due to reaching his retirement age.

<sup>&</sup>lt;sup>21</sup>See Table 2.

c. A person lost during the year prior to the recorded RY was lost due to reaching his retirement age if the difference between the year an individual joined the forces and his actual loss year was found to be 20.<sup>22</sup>

<sup>22</sup> See Table 2: Difference between entries six and three.

### IV. ANALYSIS OF THE RAW DATA

### A. OBSERVED SAMPLE SIZES AND DURATION OF OBSERVATIONS

As the purpose of this study is to condition the retention behavior of the observed group of officers on their respective SSC's, a first check should be devoted to the number of officers in each SSC. The listing in Appendix B reveals extreme differences in sample sizes. They range from zero in 16 cases 23 and one in 19 cases up to 96 for SSC xx42 for graduation year 1973. Even if the SSC's are grouped into SSC groups, 24 there remain considerable differences, as Table 3 shows.

As Table 3 already suggests, sample sizes vary also within the SSC's over the six subsequent graduation years observed. It was found that, as extremes, sample sizes varied for

- SSC xx42 between 45 and 96 officers,
- SSC xx44 between 0 and 20 officers, and
- SSC xx82 between 0 and 48 officers.

The given variation in group sizes and especially the high number of cases with sample sizes of zero or one officers will have an effect on the applicability of standard

<sup>&</sup>lt;sup>23</sup>Sample size 0: SSC xx44 and 1102 in 1970; SSC 1103 and 1308 in 1971; SSC xx44 and 1103 in 1972; SSC xx21 in 1973; SSC xx44, 1102, and 1307 in 1974; SSC xx21, xx25, xx27, and xx67 in 1975.

<sup>&</sup>lt;sup>24</sup>See Table 1.

SSC group							
xx2x			32				
xx3x	106	115	107	100	101	114	
xx4x	92	139	163	155	107	131	
xx5x	93	89	117	96	97	87	
xx6x	38	43	53	42	46	49	
xx7x	29	42	49	57	49	44	
x8xx	6	39	63	36	41	34	
xx9x	93	119	83	78	75	56	
llxx	32	43	47	36	57	51	
13xx	40						
Total	562	715	768	669	649	618	

Table 3: Number of Officers Within the SSC Groups Who Were Graduated in the Years Indicated

statistical procedures in analyzing the data and in trying to apply estimation techniques to them.

The problem is intensified by the limited amount of years k after graduation for which observations were possible.

As retention rates in the first few years after graduation can be expected to be high

- the amount of variation over the observed time period can be expected to be extremely small and thus loss rates can be expected to stay close to zero.
- The influence of chance on the variation in cases where sample sizes are as small as described can be expected to have made the observed loss rates unreliable.

### B. OVERALL LOSSES AND TRENDS

### 1. Presentation of Losses Encountered

Prior to analyzing losses by subspecialties, overall losses for the total group of officers observed will be examined to get a feel for the actual magnitude of losses encountered and to find out about possible trends over time.

Table 4 shows in part a. the accumulated losses for the indicated years k after graduation where k equals one in the year of graduation. N denotes the number of officers who were graduated in the indicated calendar year. Part b. shows the equivalent accumulated loss rates as fraction of the losses over the respective base groups N.

									.281		ı	,	ı	ı
97		158	i	ı	ı	1	1	z	.224 .	.312	ı	1	ı	1
6		126	223	ı	1	i	ı	group 1	. 173	. 256	.279	1	1	•
æ		97	183	214	t	ı	1	base g	.132 .1	. 197 . 2	.237 .2	.275	1	ı
7		74	141	182	184	ı	1	the b		. 137	.165 .2	. 220 .	.245	ı
9	lost	40	86	127	147	159	ł		16.071				.166 .2	163
ഹ	İ	6	65	90	102	108	100	fraction of	910. 60	13 .091	711. 13	72 .153		
4	officers	2	6	47	48	46	54	a fra	5 .009	7 .013	5 .061	4 .072	170. 9	,
က	of	m	S	4	16	17	25	as	.005	.007	.005	.024	.026	. 70
ч	number	-	0	0	7	10	14	losses	.002	٥.	٥.	.002	.015	,
-	Ł	0	0	0	0	0	7		0.	0.	0.	•	•	•
Z	accumulated	562	715	168	699	649	618	accumulated	562	715	168	699	649	;
l year	a. ac	1970	1971	1972	1973	1974	1975	b. ac	1970	1971	1972	1973	1974	
Grad	. I	-	7	_	7	-	•- <b>•</b>		•	٠,	•	•	•	

Accumulated losses in the graduation-year groups 1970 to 1975 for years  $k = 1, 2, \ldots, 10$  after graduation (k = 1 for the year of graduation). Table 4:

# 2. Trend Relative to Years After Graduation

velopment of the losses relative to the years k after graduation for all graduation years i. Figure II shows the unaccumulated loss rates (RL<sub>i,k</sub>) for graduation years i = 1971, 1973, and 1975 as examples. This figure also includes a curve showing the unaccumulated loss rate development over years k after graduation regardless of the graduation year (RTL<sub>k</sub>). It is calculated as

(7) RTL<sub>k</sub> = 
$$\frac{\sum_{i=1970}^{m} L_{i,k}}{\sum_{i=1970}^{m} N_{i,k}}$$
 for each year k,<sup>25</sup>

where

L<sub>i,k</sub> = loss of graduation-year group<sub>k</sub> in year k,
N<sub>i</sub> = size of graduation-year group<sub>i</sub>

m = 
$$\begin{cases} 1975 & \text{for } k = 1, ..., 5; \\ \\ 1979 - k + 1 & \text{for } k = 6, ..., 10 \end{cases}$$

All curves show a rapid increase in losses up to the 5th year after graduation. After that year, no clear trend is determinable. The peak in the 5th year is explainable by the vanishing effect of the Minimum Service Requirement (MSR).

 $<sup>^{25}</sup>k = 1$  for the year of graduation

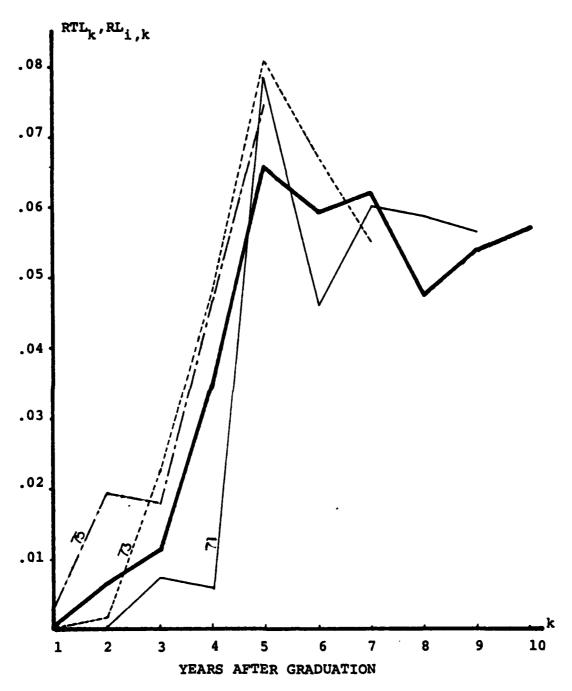


FIGURE II. Development of Unaccumulated Loss Rates Over Time (k = 1 for the year of graduation).

Shown are the curves for graduation years 71, 73, and 75 and as bold-faced curve the RTL curve according to equation (7).

Officers assigned to postgraduate education have to serve on active duty for a period of three years for the first year of education and one more year for every year thereafter upon completion of the program. As follows from the description of the SSC's selected for this study, almost all of the officers observed can be expected to have been graduated from NPS. Most of the curricula requires MSRs of at least four years. Thus the steep increase of losses in the fifth year could be a result of the cessation of the MSR obligation.

For further clarification of the observed development of the losses, it was found to be helpful to distinguish between

- a. observable losses due to the fact that officers reach their retirement age, denoted by  $LRo_{\star}^{28}$  and
- b. losses due to other reasons, denoted by LO.

The data available allowed for an extraction of the LRo and the LO. Both rates could now be calculated as respective losses over the base groups  $N_i$ . Figure III shows the unaccumulated rate for the LRo-losses, denoted by  $\mathrm{RLRo}_k$ , and the unaccumulated rate for the LO-losses, denoted by  $\mathrm{RLO}_k$ . All rates are calculated according to equation (7) with the appropriate loss type in the numerator.

<sup>&</sup>lt;sup>26</sup>OPNAVNOTE 1520, 25 June 1979, Paragraph 6

See OPNAVNOTE 1520, Enclosure 1

<sup>28</sup> See assumptions about the date of retirement in paragraph III. B.

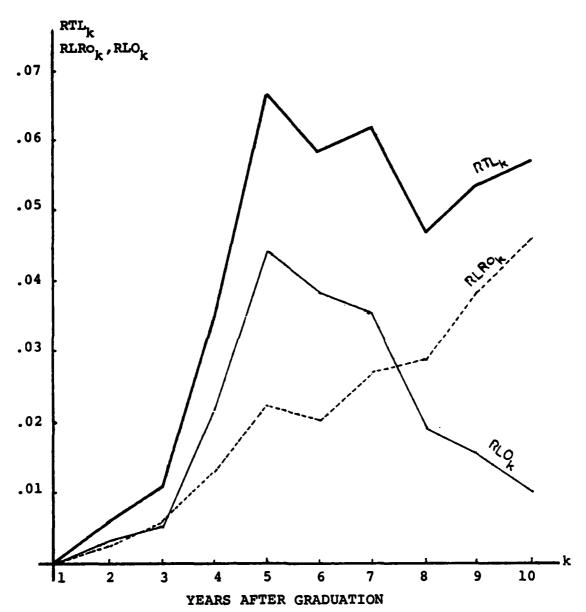


Figure III. Development of rates for total loss (RTL<sub>k</sub>), retirement loss (RLRo<sub>k</sub>) and loss due to other-than-retiremend reasons (RLO<sub>k</sub>) over time (k = 1 for the year of graduation). All rates are calculated according to equation (7) with the appropriate loss type in the numerator.

Figure III already suggests the increasing importance of retirement losses relative to total losses. This relation is emphasized in Figure IV. From the 8th year after graduation on, more than 50% of the yearly losses are accountable as retirement losses. With the 10th year, the relative importance of RLRo has reached 81%. The RLRO $_{\rm k}/{\rm RTL}_{\rm k}$  ratio for the year k = 1 could only be calculated on the basis of two out of 3981 cases.

The trend depicted in Figures III and IV and the likelihood of a steady increase of the relative importance of RLo for k greater than ten are essentially attributable to two facts:

- a. Officers who have only a few years of service left before reaching the 20-year limit--in case this limit applies to them--assess the value of the pension obtainable high enough not to leave deliberately.
- b. With increasing distance from the graduation year, the age of the officers will naturally increase. Thus the share of officers who will have to retire will increase until it approaches 100%.

The low 34% share of retirement losses in the 5th year after graduation is a result of increased losses due to other reasons, described earlier. Three major conclusions for establishing estimation and prediction models can be drawn from the facts so far known:

a. Both losses, the observable retirement loss (LRo) as well as the loss due to other reasons (LO), are not a linear

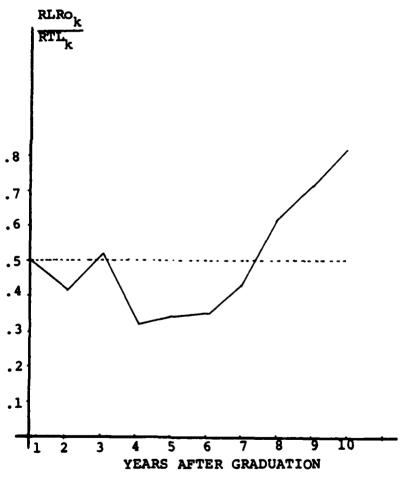


Figure IV. Development of the relative importance of retirement losses (ratio RLRo<sub>k</sub>/RTL<sub>k</sub>) over time (all graduation-year groups combined).

function of time (k) after graduation. Thus models requiring linear relationships are not applicable.

- b. The relation between total losses (L) and retirement losses (LRo) depicted in Figures III and IV suggest that estimation and prediction efforts have to be concentrated on the first few years after graduation.
- c. As losses of officers which occur because those officers retire are known deterministically, they do not have to be estimated. They are given. 29

## 3. Trend Relative to the Year of Graduation

With respect to trends relative to year i of graduation,

Table 4 indicates a generally increasing trend in the total

accumulated losses observed.

In observing retention behavior of regular Marine Corps Officers, McAfee found that the probability that an officer still stays in the system k years after entrance is the same independent of the entrance-year group the officer belonged to. Based on this assumption, which he verified with tests of homogeneity, he was able to construct a classic prediction model for retention rates k years after entrance. The probability he assumed to be stationary corresponds to the complement of the accumulated total loss rate of this study.

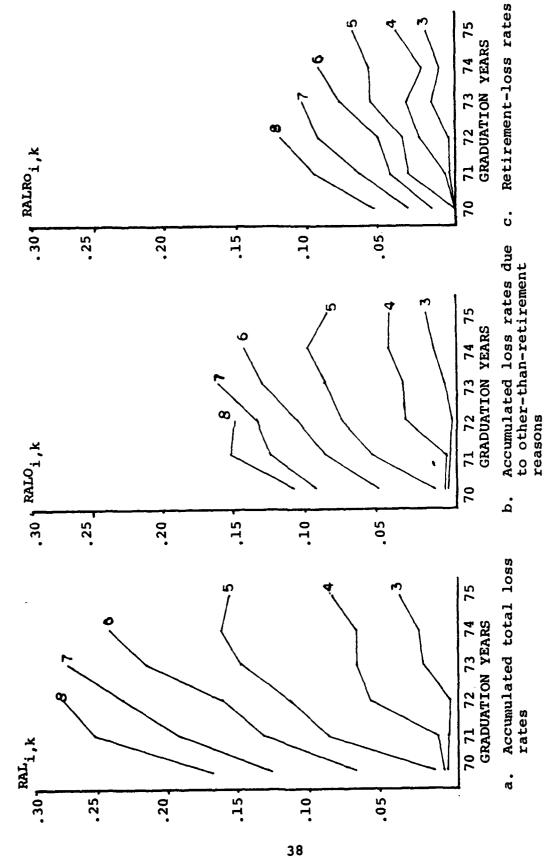
The retirement loss observed (LRo) is not identical with the retirement loss which could be calculated once a group of officers has entered the system. Part of the group will be lost due to other reasons before those officers reach their retirement age. Implications will be shown later.

<sup>30</sup> McAfee, C. K., A Cohort Model for Predicting Retention of Regular Marine Corps Officers, MS Thesis NPS, Monterey, 1970.

The data underlying this study do not suggest stationarity for the accumulated total losses with respect to subsequent graduation-year groups. For the years after graduation k = 3, 4, 6, 7 and 8, accumulated total loss rates  $(RAL_{i,k})$  are increasing over the whole range of subsequent graduation years i. Only for k = 5 do the RAL<sub>i,k</sub> decrease from 1974 as graduation year i on. Accumulated loss rates due to other-than-retirement reasons (RALO; k), as one part of the total losses, show the same trend for k = 3, 5, 6, and 7. However, the increase occurs with a smaller rate of change and the decrease for k = 5from graduation year 1974 on is more obvious. For k = 4, and 8, RALO, k decrease in the last respectively observable gradudation year, too. The trend for the accumulated retirement losses (RALRo; k), as the second group of the total losses, is increasing for all years k after graduation over the range of the subsequent graduation years i.

Figure V shows the above described developments for  $RAL_{i,k}$  in part a., for  $RAL_{0,k}$  in part b., and for  $RAL_{0,k}$  in part c. for years after graduation k = 3, 4, 5, 6, 7, and 8 over the six graduation years.

Prior to carrying out statistical tests to determine the significance of the differences among the six independent graduation-year groups for various years k after graduation, no certain conclusions about the stationary at least of the unaccumulated losses are possible. However, two summarizing remarks can be made:



Development of loss rates over graduation years for selected years after graduation  $k=3,\ 4,\ 5,\ 6,\ 7,\ and\ 8.$ FIGURE V.

- a. Accumulated losses are generally increasing for all observed years k after graduation with increasing years of graduation.
- b. The fact that accumulated retirement losses (ALRo<sub>i,k</sub>) follow this increasing trend too might be caused by two reasons:
  - Postgraduate education starts progressively later in the career of Navy officers. Thus, with increasing graduation year, the average age of the graduates is higher.
  - Progressively fewer P-coded officers pass the 20-year limit. Thus, with increasing graduation year, more officers retire after 20 years of service.

## 4. Hypothesis Testing for Stationarity

As stated in paragraph IV.B.2, retirement losses are assumed to be known. They do not have to be estimated.

Thus, in testing the significance of differences between the six graduation-year groups, the main concern has to be with the losses for other-than-retirement reasons. Tests will have to be performed for

- accumulated losses due to other-than-retirement reasons (ALO), and
- unaccumulated losses due to other-than-retirement reasons (LO).

For this purpose, Chi-square tests for independent samples have

been chosen. The tests were applied to the data according to the procedure suggested by Siegel. 31

For each year k after graduation, there is a contingency table constructable with rows representing the graduation-year groups and two columns representing the category of officers who were lost versus the category of officers who were still in service. The requirement for each of the cells within the contingency table to obtain an expected frequency of five or more for more than 80% of the cells is fulfilled for k > 3. Tests were carried out for k = 4, 5, 6, and 7.

Two hypotheses with the following Ho were tested:

- a.  $H_O$ : The graduation-year groups do not differ with respect to ALO for a given year k after graduation.
- b. H<sub>O</sub>: The graduation-year groups do not differ with respect to LO for a given year k after graduation.

As was to be expected for the accumulated loss case (ALO),  $\rm H_O$  was rejected for all k at a leve of signficance less than 0.00001. However, leaving off graduation-year groups 1970 and 1971 led to an acceptance of  $\rm H_O$  at a level of significance bigger than 0.1. That means for the latter case that if  $\rm H_O$  were rejected the error probability would be bigger than 0.1. For the unaccumulated case (LO),  $\rm H_O$  was rejected for k = 4 and k = 5 at a level of significance of less than 0.0001. However

Siegel, S., Nonparametric Statistics for the Behavioral Sciences, McGraw-Hill, New York, 1956.

 $H_{O}$  was accepted for k = 6 and k = 7 at a significance level bigger than 0.1.

The results of the Chi-square tests seem to indicate that the differences between the graduation-year groups with respect to ALO and LO are not significant at the significance levels stated when only graduation years after 1971 are considered.

However, especially when for the ALO case the development for k=6 is considered, as shown in Figure V, then the result of the Chi-square test to accept the independence hypothesis should not be valued too highly. The reason for the discrepancy between reality and test results appears to be twofold: On the one hand, the total number of cases is inflated  $^{32}$  in comparison to the actual frequencies in the loss category of the contingency table. According to Siegel, this invalidates the test. On the other hand, the test is not sensitive to trends. This can be easily recognized from the construction of the statistic.  $^{33}$ 

Using as a line fitting procedure the linear regression for RALO<sub>i,k</sub> for k=6 and using the five graduation years  $i=1970,\ldots,1974$   $X=1,2,\ldots,5$  as the carriers the fitted line equation is

<sup>&</sup>lt;sup>32</sup>See Siegel, S., p. 109.

<sup>&</sup>lt;sup>33</sup>See Siegel, S., p. 175.

 $RALO_{i,6} = 0.0390 + 0.0234x$  with  $R^2 = 0.968.34$ 

The positive slope of the line shows the increasing trend already recognized in Figure V. The following example illuminates the magnitude of the losses: Given that 600 officers graduated each year, then the losses due to other-than-retirement reasons up to the 6th year after graduation were:

- 37 officers of the first graduation-year group,
- 80 officers of the 4th graduation-year group, and extrapolating
- 164 officers of the 10th graduation-year group.

  This trend is certainly significant.

The findings with respect to the development of losses relative to the year of graduation allow essentially two major conclusions:

- a. Any model construction for estimation and prediction of losses of P-coded officers conditioned on the graduation-year group the officer belongs to that is based on the general assumption that losses occur independently from the graduation year does not comply with reality.
- b. The trends found for accumulated losses up to and including a year k after graduation over sequential graduation years are quantifiable. Using simple linear regression with graduation years as the independent

 $<sup>^{34}</sup>$ Values for RALO, for graduation years i = 1970, ..., 1974 are contained in Tables  $^{6}$ 3(D) of Appendix D.

variable seems to be a starting point to predict accumulated losses in future years k after graduation for officers belonging to a specified graduation-year group.

# V. ESTIMATION OF LOSSES CONDITIONED ON GRADUATION-YEARS AND SUBSPECIALTY CODES

#### A. DERIVATION OF THE MODEL

In Chapter II the Bayes Estimator that will be used in estimating loss rates was derived. In the previous chapter it was shown that it is possible to exclude retirement losses from the application of the Bayesian estimation technique.

As it is desired to estimate losses conditioned on the SSC an officer has obtained, it was necessary to extract the accumulated observable retirement losses (ALRO), the accumulated losses due to other reasons (ALO), and the accumulated total losses (AL) that occur up to and including the k-th year after graduation

- for each graduation-year group and within those
- for each of the 41 SSC's.

Thus the numeric losses  $AL_{i,j,k}$ ,  $ALO_{i,j,k}$ , and  $ALRO_{i,j,k}$  were obtained, where the subscripts

- i = 70, ..., 75 stand for the graduation-year observed
- j = 21, ..., 1308 stand for the 41 SSC's included, and
- $k = 1, 2, ..., k_{79}$  stand for the years k after graduation with  $k_{79}$  being the k reached in calendar year 79.

In the following derivation of the model the subscripts will be omitted and it is understood that the losses refer to a certain graduation-year i, a certain year k after graduation, and a certain SSC j. Assuming now that ALO and ALRO are

mutually exclusive, then

(8) AL = ALO + ALRO for each i,j,k.

As was already mentioned in paragraph 4.2.2. ALRO is not identical with the retirement loss (ALR) obtainable by calculating from the personal data of each individual the year k after graduation when he would reach his retirement age.

The relation between ALR and ALRO can be described by

(9) 
$$ALRO = ALR - (ALO \land ALR)$$

where  $(ALO \land ALR)$  is the retirement loss that would have been observed in addition to ALRo for year k after graduation had some officers not been lost due to other-than-retirement reasons prior to reaching their retirement age in year k after graduation. Thus equation (8) becomes

(10) AL = ALO + ALR - (ALO ↑ ALR).
Calculating for each i,j,k total accumulated loss rates (r)
from the left side of equation (8) yields

$$(11) r = \frac{AL}{N}$$

where N is the number of graduates belonging to graduation year i and to SSC j.

Calculating r from the right side of equation (8) yields

(12) 
$$r = \frac{ALO + ALRO}{N}$$

which can be algebraically manipulated and rewritten to

(13) 
$$r = \frac{ALO}{N} \times N + \frac{ALRO}{(N - ALO)} \times (N - ALO)$$

Equation (13) gives a basis for application of the Bayes
Estimation Technique to the losses due to other-than-retirement
reasons (ALO) and for including external given retirement
losses. 35

Let

$$r_{ALO} = \frac{ALO}{N}$$
 and

$$r_{ALR} = \frac{ALR_0}{(N - ALO)}$$

then equation (13) can be rewritten as

(14) 
$$r = r_{ALO} + r_{ALR} (1 - r_{ALO})$$

Assuming independence between retirement losses and losses due to other-than-retirement reasons, equation (14) is the probability statement corresponding to equation (10), where

r = P (an officer belonging to i and j is lost by year k) and  $r_{ALO}$  and  $r_{ALR}$  are the corresponding probabilities for the loss reasons described by ALO and ALR.

An alternative way to reach the result shown in equation (14) by making use of conditional probabilities is shown in Appendix C. Now, instead of the actual observed  $r_{ALO}$  the Bayesian estimate  $b_{ALO}$  will be used. Thus equation (14) can be rewritten to obtain the equation for the estimate (b) for

 $<sup>^{35}\</sup>mbox{For this study ALR}$  were not given externally, but they are calculable by applying  $\mbox{r}_{ALR}$  to N.

the accumulated total losses:

(15) 
$$b = b_{ALO} + r_{ALR} (1 - b_{ALO})$$

where the estimates and rates have to be calculated for each graduation year i and within it for each SSC j to get the loss estimate for each observable year k after graduation.

#### B. DERIVATION OF THE ESTIMATOR FOR THE MODEL

According to the definition of paragraph II.C. the estimate  $b_{\rm ALO}$  will now be derived and combined with the retirement loss  $r_{\rm ALR}$  as stated in equation (15) of the previous chapter. Let

- m<sub>i,h</sub> denote the number of officers belonging to
   Subspecialty-code group h<sup>36</sup> having graduated in
   graduation year i;
- n; j denote the number of officers belonging to SSC j, where j is one of the SSC's belonging to SSC group h who were graduated in the same year i;
- z<sub>i,h,k</sub> denote the number of officers belonging to m<sub>i,h</sub>
   who have been lost up to and including year k
   after graduation for other than retirement reasons;
   and let
- $y_{i,j,k}$  denote the number of officers belonging to  $n_{i,j}$  who have been lost up to and including year k

<sup>&</sup>lt;sup>36</sup>There are 10 SSC groups h as specified in paragraph IV.A.

after graduation for other than retirement reasons. The Bayesian estimate for the loss rate  $r_{ALO;i,j,k}$ , where

$$r_{ALO;i,j,k} = \frac{y_{i,j,k}}{n_{i,j}}$$

for each i, j, k as specified in paragraph V.A will be calculated according to equation (6) in paragraph II.C as

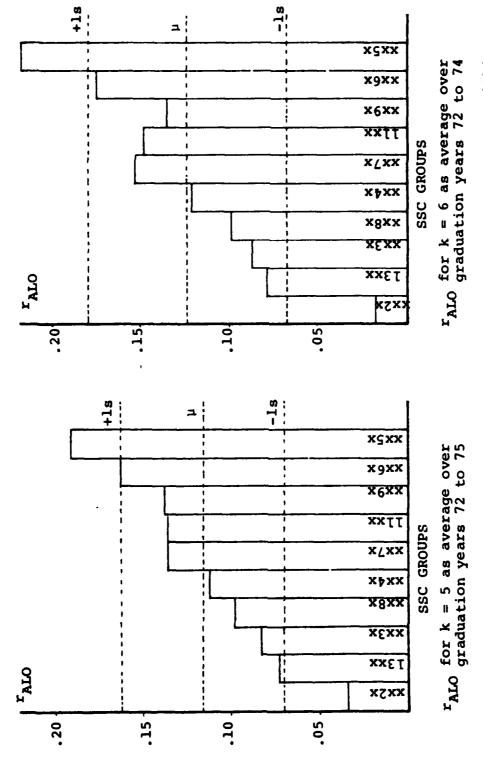
(16) 
$$b_{ALO;i,j,k} = \frac{z_{i,j,k} + y_{i,j,k}}{m_{i,h}}$$

for each i, j, k as specified in paragraph V.A and each h as specified above. The calculation of the estimate  $b_{ALO}$  is possible as long as  $m_{i,h}$  is not empty.<sup>37</sup>

The construction of the Bayesian estimate, as explained in Chapter II, would have allowed for including the total group of graduates of a graduation year i as a first refining sample into the estimator. However, this would have the undesirable effect that this inflated sample would almost completely cover up basic differences in the retention behavior between officers belonging to different SSC groups and to different SSC's. The existence of differences among the SSC groups is shown in Figure VI.

The first histogram shows the accumulated losses due to other-than-retirement reasons (ALO) up to year k=5 as average over graduation years 1972 to 1975. The SSC groups

 $<sup>^{37}</sup>$ For the six graduation years (i = 1970, ..., 1975) and for all SSC groups (h = 1, ..., 10)  $m_{i,h}$  was found not to be empty.



year k after graduation as average over the indicated graduation the Subspecialty-code groups up to and including the indicated Accumulated losses due to other-than-retirement reasons within years. Figure VI:

have been ordered in terms of ascending loss rates. The second histogram shows the same losses up to year k=6 as averages over graduation years 1972 to 1974. The order of the SSC groups remained the same as in the first histogram. In addition, the histograms show the mean losses and lines bounding the region of  $\pm$  1 standard deviation s. Besides minor changes in the order of those groups with losses near the mean ( $\mu$ ), the ranking in terms of ascending loss rates stays stable. Groups  $\pm$  2x and  $\pm$  3x and  $\pm$  3x and  $\pm$  4x and groups 13x and  $\pm$  4x and groups to the border lines of  $\pm$  1s.

In order to test the significance of the differences among the SSC groups, again Chi-square tests were applied as introduced in paragraph IV.B.3.

The hypotheses

 $H_0$ : The SSC groups do not differ with respect to their ALO rates for k = 5;

 $H_O$ : The SSC groups do not differ with respect to their ALO rates for k = 6;

were both rejected at a significance level of less than 0.00001. For the calculation of the retirement losses  $r_{ALR;i,j,k}^{39}$  which is the second entity needed to estimate the overall loss-rate estimate  $b_{i,j,k}$  two assumptions have to be made:

 $<sup>^{38}</sup>$ Last year of observation was year 1979. There were no data available for  $k \approx 6$  for graduation year 1975 (corresponds to calendar year 1980).

 $<sup>^{39}</sup>$ Subscripts have been added to  $r_{\mbox{\scriptsize ALR}}$  as specified above.

Let

ui,j,k denote the number of officers belonging to SSC j of graduation year i who retired up to and including year k after graduation.

Then, as was shown in paragraph V.A.

(17) 
$$r_{ALR;i,j,k} = \frac{u_{i,j,k}}{n_{i,j} - y_{i,j,k}}$$

for any i, j, k as specified in chapter V.A.

Now, if  $n_{i,j}$  equals  $y_{i,j,k}$  for any k than  $u_{i,j,k}$  will be zero and thus  $r_{ALR;i,j,k}$  has to be assumed to be zero for the SSC j involved. This means that if everyone belonging to  $n_{i,j}$  was lost up to year k after graduation due to other-than-retirement reasons, nobody of that group could have retired.

The second assumption is necessary for the case that nobody belonging to a certain SSC j graduated in a certain graduation year i. In this case  $n_{i,j}$  is zero for that SSC j in the specific year i and  $u_{i,j,k}$  and  $y_{i,j,k}$  are zero for all k and for the specific j and i under consideration. However, the Bayesian estimator allows for calculating the estimate  $b_{ALO}$  as long as at least  $m_{i,h}$  is not zero too, as was shown earlier. For calculating the retirement rate, it was assumed that if  $n_{i,j}$  were empty it would have the same retirement rate as the SSC group  $m_{i,h}$  it belongs to.  $^{41}$ 

u, corresponds to ALRo for the graduation year i, the SSC j and the year k after graduation under consideration.

This method was mentioned by Prof. R. Weitzman in his lectures at NPS and is known as "Incomplete Tree Method."

#### C. PRESENTATION AND DISCUSSION OF RESULTS

### 1. Presentation of Results

Appendix D shows in six sections—one section for each graduation year—loss rates and estimates of the rates conditioned on the graduation year and the Subspecialty code an officer has obtained.

The graduation-year sections contain the following tables:

- Table 1(D): Accumulated total loss rates  $(r_{i,j,k})$  and their respective estimates  $(b_{i,j,k})$  for each SSC j and each year k after graduation for the graduation year i of that section.
- Table 2(D): Accumulated loss rates for other-thanretirement losses (r<sub>ALO;i,j,k</sub>) and their
  estimates (b<sub>ALO;i,j,k</sub>) for the same i, j,
  and k as in Table 1(D).
- Table 3(D): r<sub>i,h,k</sub>, r<sub>ALO;i,j,k</sub> and r<sub>ALR;i,h,k</sub> for the ten SSC groups h; RAL<sub>i,k</sub>, RALO<sub>i,k</sub>, and RALR<sub>i,k</sub> for the total group of graduates in that graduation year i and for all years k after graduation.

#### 2. Discussion of Results for Selected Subspecialty Codes

As it seemed impossible to exhaustively discuss the results for all 41 Subspecialty codes involved, three of the ten SSC groups with their Subspecialty codes were selected according to their special appearance in Figure VI of Chapter

#### V.B. These are:

- SSC group xx2x which shows low loss rates and extremely small sample sizes of the SSC involved,
- SSC group xx5x which shows the highest loss rates and also high sample sizes of the SSC belonging to it, and
- SSC group xx4x which shows loss rates closest to the mean loss rate of the ten SSC groups and which also shows high sample sizes of the SSC involved.

#### a. Subspecialty Code Group xx2x

SSC group xx2x seems to be one of the groups that caused the rejection of the equivalence tests in paragraph V.B. It is the group with the lowest loss rates and with the lowest sample sizes of the S\$C's belonging to the group.

Table 5 shows the magnitude of the actual accumulated losses due to other-than-retirement reasons (ALO) for graduation years 1972 to 1975 up to and including the 5th year, the 6th year, and the 7th year after graduation (k equals one in the year of graduation).

A total of four officers out of 119 graduates were lost up to and including the 5th year after graduation and only two out of 106 graduates were lost up to and including year six after graduation.

These extremely small losses do not allow for recognition of any trends over the graduation years or differences among the Subspecialty codes. Table 6 shows the group sizes of all SSC's in group xx2x, and Table 7 shows the accumulated

Graduation Year	Size of group xx2x		cers lost () and include k =		from SSC	SSC size	
		5	6	7			
			~~~~~				
1972	32	1	1	1	xx21	3	
1973	34	Q	0	0		-	
1974	40	1	1	-	xx22	10	
1975	13	2	-	-	xx24	7	

Table 5: Accumulated losses due to other-than-retirement reasons (ALO) in SSC group xx2x for k = 5, 6, 7. (Last year of observation is 1979. k = 1 in the year of graduation.)

SSC Grad year	xx21	xx22	xx23	xx24	xx25	xx26	xx27
1972	3	9	3	5	6	5	1
1973	0	11	4	11	2	4	2
1974	3	10	3	15	1	7	1
1975	0	4	1	7	0	1	0

Table 6: Sample sizes of Subspecialty codes in group xx2x for graduation years 1972 to 1975.

SSC			Graduation years							
		1972		197	73	197	4	19	75	
	AL	2	AL	ALO	AL	ALO	AL	ALO	AL	
xx21	r	.333	.333			.0	.0			
	b	.057	.057	.0			.023			
xx22	r	.0	.0	.0			.200			
							.147			
xx23					.250		.0			
							.023			
							.0			
	b	.027	.027	.0	.0	.018	.018	.200	.520	
xx25	r	.0	.0	.0	.0	.0	1.0			
							1.0			
							.0			
	b	.027	.222	.0	.0	.021	.021	.143	.143	
xx27							.0			
							.024			

Table 7: Accumulated loss rates due to other-than-retirement reasons  $(r_{ALO})$  and their estimates  $(b_{ALO})$  and total accumulated loss rates  $(r_{AL})$  and their estimates  $(b_{AL})$  for k=5.

loss rates due to other-than-retirement reasons  $(r_{ALO})$  and their estimates  $(b_{ALO})$  as well as the accumulated total loss rates (r) and their estimates (b) for the fifth year after graduation.

As was desired, the estimate rows do not show empty cells. Instead, they show group averages according to the construction of the Bayesian estimator. The smoothing effect of the estimation technique is due to the fact that the estimate is a compromise between the information gained from the group behavior and the information gained from the behavior within a specific SSC. 42 The effect of chance influences on extremely small samples like the SSC's in this group is clearly visible in the case of SSC xx24. Up to the 5th year after graduation there is no loss in graduation-year groups 1972 to 1974. Graduation-year group 1975, however, not only happens to be smaller than the two previous ones, but this group loses two officers for other-than-retirement reasons and two more retire. Thus, more than 50% of this group are lost within the first five years after graduation. Looking at the loss rates only, this is an alarming increase. As the total SSC group xx2x consists of only 13 officers for that graduation year, the Bayesian estimate fails to remove this obvious chance effect significantly. However, as part

<sup>42</sup> See paragraph II.C.1.

of the chance influences are caused by the retirement losses, chance effects are less dramatic for the  $b_{ALO}$  values. This supports the idea already mentioned in the context of analyzing general trends in paragraph IV.B.2 to base prediction models on the  $b_{ALO}$  values and insert known future retirement losses in the way described by the model derived in paragraph V.A.

## b. Subspecialty-code group xx5x

SSC group xx5x is the group with the highest loss rates for k=5 as well as for k=6 in Figure VI. Together with groups xx3x and xx4x, it is also a group with high group sizes  $m_{i,h}$  relative to the other groups. Its losses—as shown in Figure VI—exceed the region bounded by  $\pm$  1 standard deviation significantly.

Table 8 shows the sizes  $n_{i,j}$  of the SSC's in this group for graduation years 1972 to 1975.

SSC					
Grad year	xx51 <sup>′</sup>	xx52	xx54	xx55	xx56
1972	11	19	54	27	6
. 1973	11	15	38	26	6
1974	9	18	28	31	11
1975	12	17	33	18	7

Table 8: Sample sizes of Subspecialty codes in group xx5x for graduation years 1972 to 1975.

Within this group, no extreme low SSC sizes are depicted. However the SSC's vary significantly in size. Table 9 shows again the accumulated loss rates  $r_{\rm ALO}$  and r as well as their estimates  $b_{\rm ALO}$  and b.

ssc				Graduat	tion-yea	ars			
		19	72	19	73	19	74	19	75
		ALO	AL	ALO	AL	ALO	AL	ALO	AL
xx51	r	.0	.0	.0	.0	.222	.222	.083	.083
	b	.102	.102	.112	.112	.142	.142	.091	.091
<b>x</b> x52	r	.105	.105	.267	.267	.056	.056	.118	.118
	þ	.110	.110	.144	.144	.122	.122	.096	.096
жж54	r	. 204	.222	.184	.237	.143	. 286	.121	.303
	þ	.140	.160	.142	.197	.136	.280	.100	.286
<b>x</b> x55	r	.0	.037	.038	.192	.194	.290	.056	.222
	b	.090	.124	.107	.250	.148	.251	.086	.247
<b>xx</b> 56	r	.0	.333	.0	.0	.0	.091	.0	.143
	b	.106	.404	.118	.118	.120	.200	.085	.216

Table 9: Accumulated loss rates due to other-than-retirement reasons  $(r_{ALO})$  and their estimates  $(b_{ALO})$  and total accumulated loss rates  $(r_{AL})$  and their estimates  $(b_{AL})$  for k=5.

Again the "smoothing" effect of the estimation technique can be seen. And again the idea is supported that prediction models should be based on the  $b_{ALO}$  values as carriers rather than on the b values: For SSC xx54, which is the SSC with the highest sizes  $n_{i,j}$ , the b values indicate a steady increase of losses from graduation year 1972 to 1975. However, looking at the  $b_{ALO}$  values, the rates  $r_{ALO}$  and their estimates  $b_{ALO}$  decrease. Thus, only the relative number of officers who had to retire increased for the subsequent graduation year groups. For SSC xx56, retirement losses are the only cause of variation over the subsequent graduation years.

#### c. Subspecialty-code group xx4x

SSC-group xx4x is the group for which the accumulated loss rates for k=5 and k=6 shown in Figure VI lie closest to the mean rates of the ten Subspecialty-code groups. It is also the group with the highest group sizes  $m_{i,h}$  between graduation years 1971 and 1975. Table 10 shows the sizes  $n_{i,j}$  of the SSC in this group for graduation years 1972 to 1975.

SSC xx4x not only shows the most extreme differences in the SSC sizes but also contains the SSC's with the most extreme differences in terms of curricula at the NPS as Table 11 shows. However, these differences do not cause correspondingly great differences among the loss rates of the various SSC's within the SSC group xx4x, as Table 12 shows.

<sup>43</sup> The fact that retirement losses show an increasing trend with increasing graduation-yeasr was already shown in paragraph IV.B.2.

1973     96     0     23     36       1974     54     0     25     28	SSC Grad year	xx42	**44	xx48	xx49
1974 54 0 25 28	1972 ,	85	Q	34	44
	1973	96	0	23	36
1975 57 20 30 24	1974	54	0	25	28
	1975	57	20	30	24

Table 10: Sample sizes of Subspecialty codes in group xx4x for graduation years 1972 to 1975.

SSC	Subspecialty Title	Curriculum
xx42	Operations Research/Systems Analysis	360
xx44	Anti-Submarine Warfare Systems Technology	525
**48	Meterology	372
xx49	Oceanography	440

Table 11: Subspecialty codes with their title and Curricula at NPS Source: OPNAVNOTE 1520, 25 June 1979

SSC				G	raduation	on years	s		
		19	72	19′	73	19	74	19	75
		ALO	AL	ALO	AL	ALO	AL	ALO	AL
xx42	r	.071	.094	.063	.135	.148	.167	.140	.228
	b	.089	.112	.064	.137	.118	.137	.101	.193
xx44	r							.0	.0
	þ	.098	.123	.065	.129	.103	.121	.073	.073
xx48	r	.176	.206	.043	.130	.080	.120	.067	.100
	b	.112	.143	.062	.147	.098	.138	.081	.114
<b>x</b> x49	r	.091	.114	.083	.111	.036	.036	.042	.083
	b	.097	.119	.068	.096	.089	.089	.077	.118

Table 12: Accumulated loss rates due to other-than-retirement reasons  $(r_{ALO})$  and their estimates  $(b_{ALO})$  and total accumulated loss rates  $(r_{AL})$  and their estimates  $(b_{AL})$  for k=5.

Figures VII and VIII show the differences in the variation of the loss rates and their estimates between group xx5x and group xx4x for k=5 and graduation years 1972 to 1975. Figure VII shows for each graduation year the mean of the  $r_{ALO}$  for the SSC's in group xx4x and group xx5x and the region bounded by  $\pm$  1 standard-deviation. Figure VIII shows the same for the respective  $b_{ALO}$ . In both cases it can be seen that—except for graduation year 1975—the standard deviations are bigger for group xx5x relative to group xx4x.

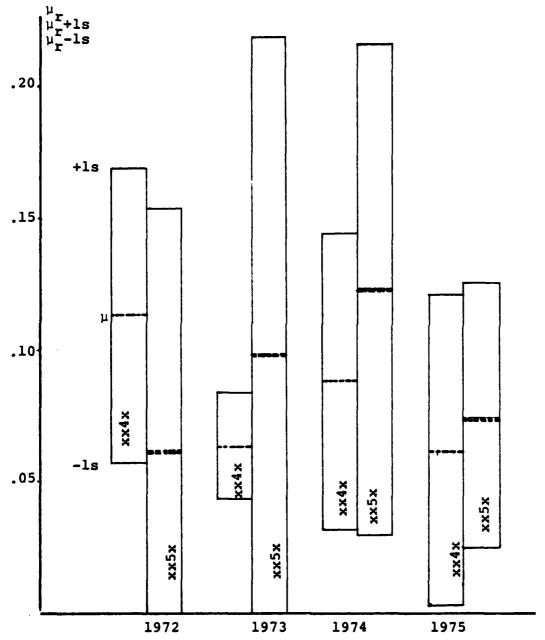


Figure VII: Means ( $\mu$ ) of the r of the SSC's in SSC groups xx4x and xx5x and the region bounded by  $\pm$  1 standard deviation (s) for k = 5 years after graduation. Shown are graduation-year groups 1972 to 1975.

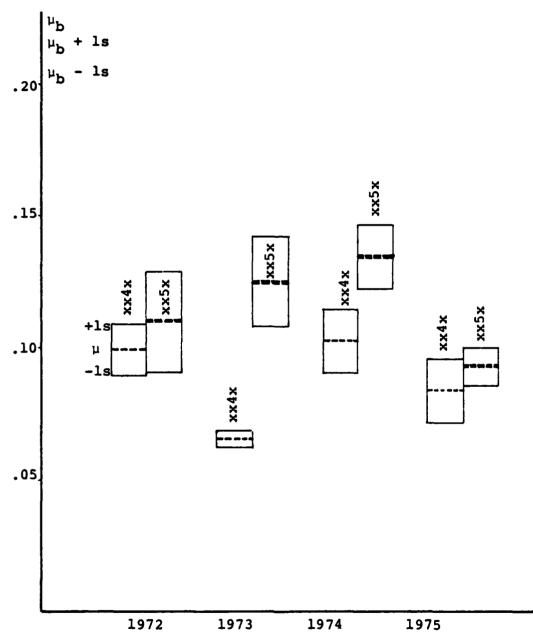


Figure VIII: Means ( $\mu$ ) of the b<sub>ALO</sub> of the SSC's in SSC groups xx4x and xx5x and the regions bounded by  $\pm$  standard deviation (s) for k = 5 years after graduation. Shown are graduation-year groups 1972 to 1975.

A comparison of the figures reveals also the effect of the Bayesian estimation technique on the differences between the losses for each SSC within the SSC group they belong to. The size of the standard deviations depicted have decreased considerably from the  $r_{\rm ALO}$  figure to the  $b_{\rm ALO}$  figure.

### 3. Correlation Between Rates and Their Estimates

The estimates calculated according to equation (15) of paragraph V.A. in conjunction with equation (16) of paragraph V.B. are supposed to provide for valid estimates of the loss rates conditioned on the graduation year and the SSC. Provided there exists a linear relationship between the estimates b and the actual rates  $r_{\ell}$  the Pearson correlation coefficient  $(r_{\rm br})$  should be an indication for the closeness of the estimates to the actual rates.

The coefficient is calculated according to the following equation:

(18) 
$$r_{br} = \frac{L \sum_{\ell=1}^{L} b_{\ell} r_{\ell} - \sum_{\ell=1}^{L} b_{\ell} \sum_{\ell=1}^{L} r_{\ell}}{\sqrt{L \sum_{\ell=1}^{L} b_{\ell}^{2} - (\sum_{\ell=1}^{L} b_{\ell})^{2}} \sqrt{L \sum_{\ell=1}^{L} r_{\ell}^{2} - (\sum_{\ell=1}^{L} r_{\ell})^{2}}}$$

where  $-1 < r_{br} < +1$ ,

L = total number of estimates =
 total number of rates.

The graphs included in Appendix E depict the relation between rates and estimates for each graduation year. They do not reveal any strong pattern of deviation from linearity.

However, they show for all graduation years that for rates close to zero the estimate b tends to deviate more from the actual rate r than for higher loss rates.

Table 13 summarizes the findings for r for graduation years 1970 to 1975. In addition it shows the amount of variation of the loss rates that is explained by the estimator. denoted by  $r_{br}^2$ . The total number of estimates—L as shown in the table—is the product of the number of SSC's j and the number of years k after graduation for the specific graduation year for the specific graduation year, where SSC's of size zero are excluded for that graduation year.

In analyzing the correlation between the loss rates and their estimates it is possible to answer the question how close the behavior of officers belonging to the various SSC's follows the trend of the respective SSC groups.

			<del></del>				
		Gra	duation	years			
	1970	1971	1972	1973	1974	1975	
r <sub>br</sub>	.85	.95	.93	.91	.76	.87	
r <sub>br</sub> <sup>2</sup>	.72	.91	.86	.84	.58	.76	
L	380	360	304	273	228	185	

Table 13: Correlation-coefficients r for accumulated total loss-rates r and their estimates b and the respection. L is the number of coefficients calculated for the resp. graduation year.

According to the structure of the estimator,  $r_{\rm br}$  would equal 1.0 only if for that graduation year for each of the ten SSC groups the loss rates for all SSC's belonging to the SSC group were identical. The more  $r_{\rm br}$  deviates from 1.0 the more differences between the various SSC's and their respective SSC groups are encountered. Thus, according to the coefeficients in Table 13, this variation is highest for graduation year 1974 and extremely low for graduation year 1971.

As was mentioned before, chance influences should have more impact in smaller groups than for bigger ones. Accordingly the coefficients  $r_{\rm br}$  should be smaller when only small SSC sizes are considered for calculating  $r_{\rm br}$ .

Table 13 compares the  $r_{\mbox{\scriptsize br}}$  which were obtained for SSC sizes

- bigger than 20 versus smaller than 20,
- bigger than 10 versus smaller than 10, and
- bigger than 5 versus smaller than 5.

The values of  $r_{\rm br}$  show generally an increasing trend with increasing SSC sizes used for calculated them. However, for graduation year 1970, 1971, and 1973 the values  $r_{\rm br}$  increase again when SSC sizes less than five are used for calculating them. This unexpected increase becomes explainable when only accumulated loss rates due to other than retirement reasons  $r_{\rm ALO}$  are correlated with their estimates  $b_{\rm ALO}$ . Now, not only the expected trend is true for all graduation years but the differences between the coefficients are more obvious, as Table 15 shows.

			SSC sizes selected	
Grad	year	a) GE 20 b) LT 20	a) GE 10 b) LT 10	
1970	a)	.99 (100)	.99 (140)	.82 (230)
	b)	.83 (280)	.81 (240)	.92 (150)
1971	a)	.99 ( 99)	.93 (162)	.92 (225)
	b)	.95 (261)	.96 (198)	.98 (135)
1972	a)	.97 (112)	.95 (160)	.96 (216)
	b)	.92 (192)	.92 (144)	.90 ( 88)
1973	a)	.99 ( 84)	.97 (133)	.90 (175)
	b)	.90 (189)	.89 (140)	.92 ( 98)
1974	a)	.99 ( 72)	.96 (114)	.94 (168)
	b)	.73 (156)	.71 (114)	.68 ( 60)
1975	a)	.97 ( 65)	.94 (105)	.92 (130)
	b)	.86 (120)	.86 ( 80)	.85 ( 55)

Table 14: Comparison of correlation coefficients r for accumulated total loss rates r and their estimates b under consideration of different SSC sizes. (Number of cases in parentheses)

			SSC sizes selected	
Grad-	year	a) GE 20 b) LT 20		a) GE 5 b) LT 10
1970	a)	.98	.98	.75
	b)	.64	. 56	.48
1971	a)	.98	.89	.74
	b)	.60	.45	.26
1972	a)	.92	.89	.83
	b)	.44	.06	.06
1973	a)	.98	.94	.74
	b)	.51	.23	.25
1974	a)	.98	.90	.82
	b)	.42	.40	.50
1975	a)	.91	.86	.81
2-,-	b)		.36	.24

Table 15: Comparison of correlation-coefficients  $r_{\text{ALO}}^{\text{b}}_{\text{ALO}}$  for loss-rates  $r_{\text{ALO}}$  and their estimates  $r_{\text{ALO}}$  for different SSC sizes.

(Number of cases is the same as in Table 14.)

Obviously, the unexpected increase of  $r_{\rm br}$  for SSC-sizes of less than five individuals was caused by the effect of the retirement rate  $r_{\rm ALR}$ , which enters the model as rate and not as estimate. The resulting adjustment in the direction of the observed overall loss rate is relatively bigger for small SSC sizes, as was to be expected.

The extremely low value of  $r_{\rm br}$  in Table 15 for graduation year 1972 and SSC sizes less than ten is caused by the following facts: 19 out of the 41 SSC's did not show any losses due to other-than-retirement reasons (ALO) over the whole observed time up to year k=8 after graduation. This resulted in 162 cases with  $r_{\rm ALO}$  equal to zero. Out of the 144 cases for SSC sizes of less than 10 individuals, 44 138 are cases with  $r_{\rm ALO}$  equal to zero.

Thirty two of the 138 cases which represent the most extreme differences between  $r_{ALO}$  and  $b_{ALO}$  and thus contribute heavily to the low  $r_{br}$  values are shown in Table 16. The SSC's shown are the only ones out of their respective SSC groups which have zero ALO losses over the whole observed period and they happen to be also the only ones out of their groups with SSC sizes less than ten. In addition group xx2x consists completely of SSC's with sizes less than ten. Its seven SSC's make up 56 cases out of which 50 show  $r_{ALO}$  values of zero. The six remaining cases have an  $r_{ALO}$  value of .333.

 $<sup>^{44}</sup>$ See Table 14: 18 SSC with less than 10 officers x 8 observed years k.

ssc	k =								
	r <sub>ALO</sub>	.0	.0	.0	.0	.0	.0	.0	.0
	r <sub>ALO</sub>								
	r <sub>ALO</sub>								
1102	r <sub>ALO</sub>								

Table 16: Comparison rates of  $r_{ALO}$  and their estimates for four selected SSC of the group of 18 SSC of graduation year 1972 which consist of less than ten individuals each.

The respective  $b_{ALO}$  values range from .0 to .03. Group xx3x--consisting of five SSC's--contributes 3 SSC's to the group of SSC's with less than ten individuals. Each of the three does not show any ALO losses within the eight observed years, whereas the two remaining SSC's show  $r_{ALO}$  values of up to .113 for k=8 and SSC xx31. This results in differences between the  $r_{ALO}$  values and their respective estimates of up to .083. The same reasoning applies to SSC group 13xx, which consists of seven SSC's. Three of them belong to the class of SSC's with less than ten individuals, and all three do not show ALO losses. The losses encountered for the rest of the SSC's in group 13xx result in differences between  $r_{ALO}$  values and their estimates of up to .71.

However, it has to be stated that the class of SSC's having sizes of less than ten individuals each represents only 67 out of the 768 officers who graduated in 1972. Up to the eighth year after graduation, which was the last year observed, only one of the 67 officers was lost due to other-than-retirement reasons. From the remaining 701 officers belonging to SSC's with sizes of ten and more individuals, 119 were lost due to the same reasons.

Except for group xx2x the trends in the SSC groups are results of losses occurring outside the class of SSC's with less than ten individuals. These group trends cover almost completely the fact that there occurred no loss in the small-size class of SSC's except the one in group xx2x.

Thus, the  $b_{ALO}$  values for those small SSC's are more a result of the respective SSC-group trends than of the actual losses within the SSC's. Correpondingly, for SSC's with sizes of less than ten individuals the actual group losses due to other-than-retirement reasons correlate with a coefficient of .9942 with the estimates  $b_{ALO}$  of the SSC losses whereas the  $r_{ALO}$  values for the actual individual losses correlate only with the coefficient of .06 with their estimates  $b_{ALO}$ . These extreme results shown above for graduation year 1972 could not be found for the rest of the graduation-years.

Thus, it cannot be assumed that smaller SSC sizes tend to have smaller loss rates or that for smaller SSC sizes the Bayesian estimator is less valid. Rather, it has to be assumed that the extreme low losses in small-size SSC for graduation year 1972 were a result of chance influences and that the Bayesian estimator achieved what it was supposed to achieve, namely to cope with non-system-inherent chance influences mainly on small-sized groups by adjusting the observed values in the direction of predominant trends.

## VI. SUMMARY OF CONCLUSIONS AND RECOMMENDATIONS

#### A. CONCLUSIONS

The distinction between retirement losses and losses for other-than-retirement reasons that formed the basis for the construction of the estimation model was most helpful in analyzing underlying trends of the observable losses with respect to the graduation-year group an officer belonged to and with respect to the years after graduation. It led to the observation that the estimation and prediction effort can be restricted to a relatively short-time period of approximately ten years after graduation. The data showed that with the tenth year after graduation the relative share of observable retirement losses with respect to total losses has already reached 81%, with a strongly increasing trend. The data indicated also that losses due to other-than-retirement reasons do not vary linearly with subsequent years after graduation. They increase rapidly to a peak level between year five and year seven after graduation. Then they decrease steadily.

In addition it was found that relative to the year of graduation accumulated total losses, accumulated retirement losses, and accumulated losses due to other-than-retirement reasons varied with an increasing trend between graduation years 1970 and 1974 and--with the exception of the retirement losses--decreased again.

The ten SSC groups observed were found to differ significantly with respect to their losses due to other-than-retirement reasons. The significance of differences among the SSC's within their respective SSC groups was not testable because of limitations imposed by the available test procedures.

The extreme variations in the sample sizes among the 41 SSC's as well as for the single SSC's over time--ranging from zero to over 100 individuals--did not affect the applicability of the Bayesian estimation technique. The estimates showed the expected smoothing effect of the technique, which resulted from the fact that the value of the estimate is a compromise between the information gained from the behavior of the total SSC group and the special behavior of officers in the different SSC's within the groups. Thus the spoiling effect of chance influences especially on small-sized groups could be successfully limited.

correlating the accumulated total loss rates with their estimates resulted in coefficients between .76 for the graduation year 1974 and .95 for graduation year 1971. As long as only SSC's with sizes bigger than ten individuals each were considered for calculating the coefficients, they ranged from .94 to .99. The magnitude of the correlation coefficient does not only indicate how close the estimates follow their respective rates. It was also found to be a useful indicator for the differences in the retention behavior of officers belonging

to different SSC's within the various SSC groups. A coefficient of 1.0 would be found only in cases where either all SSC groups consisted only of one SSC each or where the loss rates for all SSC's within a SSC group were identical for all SSC groups. With regard to this interpretation, the correlation coefficients were surprisingly high.

### B. RECOMMENDATIONS FOR EXTENSIONS INTO PREDICTION MODELS

Using the estimates for the loss rates  $(b_{ALO})$  instead of the actual observed rates  $(r_{ALO})$  provides a more stable and reliable basis for carrying out predictions of future losses of officers who have already graduated and also of losses for future graduation-year groups.

Prediction could be done by using the binomial distribution in a way already proposed by McAfee, however, without his stationarity assumption, as already discussed.

- N be the number of officers belonging to SSC j who graduate in year i;
- Yi,j,k be the number of officers belonging to Ni,j who have been lost due to other-than-retirement reasons up to and including year k after graduation; and let
- pi,j,k be the probability that an officer belonging to

Let

<sup>45&</sup>lt;sub>McAfee</sub>, C. K., p. 12.

 $N_{i,j}$  belongs also to  $Y_{i,j,k}$ , estimated by  $b_{ALO;i,j,k}$ .

Then

(19) 
$$P(Y_{i,j,k} = y | N_{i,j} = n) = {y \choose n} p_{i,j,k}^{Y} (1 - p_{i,j,k})^{n-Y}$$
.

Because the stationarity assumption about  $p_{i,j,k}$  is no longer in existence, future values of  $p_{i,j,k}$  have to be predicted by projecting the corresponding  $b_{ALO;i,j,k}$  to the future. This can be done by exploiting known trends and establishing trend lines as proposed at the end of paragraph IV.B.3: For each year k after graduation and each SSC j, trends of the loss estimates  $b_{ALO}$  over graduation years i are observable. They can be translated into trend lines, for example, by using regression models. Thus, for all SSC j and years k after graduation of interest,  $b_{ALO}$  values for future graduation years i and future years k after graduation or  $b_{ALO}$  values for past graduation years i but future years k after graduation are obtainable.

Using these balo values as entries for the respective  $p_{i,j,k}$  in above equation (19) yields probability statements about future losses due to other-than-retirement losses. Probability statements about total future losses are obtainable by using an approach similar to that one proposed in Chapter V. As long as only expected values of future loss rates are wanted, the approach represented by equation (19) is not needed: The predicted  $b_{ALO;i,j,k}$  values can be combined

with the known future  $r_{ALR;i,j,k}$  values as proposed in Chapter V. Thus, expected values of future loss rates will be obtained.

# APPENDIX A-1 SUBSPECIALTY CODES

Subspecialty-code	Subspecialty title
<u> </u>	roup xx2x NATIONAL SECURITY AFFAIRS
xx21	Mideast, Africa, or South Asia
<b>x</b> x22	Far East, Southeast Asia, or Pacific Ocean
xx23	Western Hemisphere
<b>x</b> x24	Europe, USSR
<b>x</b> x25	International Organizations and Negotiations
<b>xx</b> 26	Strategic Planning
<b>xx27</b>	Nuclear Planning
<u> </u>	Group xx3x ADMINISTRATIVE SCIENCE
xx31	Financial Management
<b>xx</b> 32	Material Management
<b>x</b> x33	Manpower/Personnel Analysis
<b>x</b> x34	Logistics Management
<b>xx38</b>	Human Resource Management
g	Froup xx4x, APPLIED LOGIC AND OPERATIONS SYSTEMS TECHNOLOGY
xx42	Operations Research/Systems Analysis
xx44	Anti-Submarine Warfare Systems Technology
xx48	Meterology
<b>x</b> x49	Oceanography

Subspecialty-co	de Subspecialty Title
	Group xx5x NAVAL SYSTEMS ENGINEERING
xx51	Naval Construction and Engineering
xx52	Nuclear Engineering
xx54	Naval/Mechanical Engineering
xx55	Engineering Electronics
xx56	Underwater Acoustics
	Group xx6x WEAPON SYSTEMS ENGINEERING
xx61	Weapons Systems Technology
xx62	Chemistry
<b>x</b> x63	Weapons Systems Science (Physics)
<b>x</b> x67	Nuclear Effects (Physics)
	Group xx7x AERONAUTICAL ENGINEERING
<b>xx71</b>	Aeronautical Engineering
жх72	Aeronautical Engineering with Avionics
	Group xx8x COMMUNICATIONS
xx81	Communications Engineering
xx82	Telecommunications Systems Management
	Group xx9x COMPUTER TECHNOLOGY
xx91	Computer Science

Computer Systems Management

**x**x95

Subspecialty-code	Subspecialty Title
9	Group llxx CIVIL ENGINEERING
1101	Facilities Engineering
1102	Petroleum Engineering
1103	Ocean Engineering
9	Group 13xx SUPPLY
1301	Supply Acquisitions/Distribution Management
1302	Systems Inventory Management
1304	Material Movement
1305	Retailing
1306	Acquisition Contract Management
1307	Petroleum Management
1308	Subsistence Technology

ORIGINAL RECORD STRUCTURE

SSC	(8)
Year eligible to retire	(7)
Month/Year Year of loss to to to	(9)
	(4) (5) (6) (7) (8)
Year of gradua- tion	
h/Year Month/Year Year of Month/Year irth of entry gradua- of Minimum tion Service Retion guirement	(3)
SSN Month/Year of birth	(2)
SSN	(1)

APPENDIX B
SAMPLE SIZES FOR SUBSPECIALTY CODES

ssc	1970	1971	1972	1973	1974	1975
xx21 xx22 xx23 xx24 xx25 xx26 xx27	6 4 7 2 8 2	7 8 1 16 4 2	3 9 3 5 6 5	0 11 4 11 2 4 2	3 10 3 15 1 7	0 4 1 7 0 1
xx31	66	66	62	59	56	72
xx32	32	37	34	24	33	25
xx33	3	6	5	7	2	2
xx34	3	3	4	6	3	4
xx38	2	3	2	4	7	11
xx42	45	69	85	96	54	57
xx44	0	1	0	0	0	20
xx48	4	18	34	23	25	30
xx49	43	51	44	36	28	24
xx51	2	8	11	11	9	12
xx52	14	6	19	15	18	17
xx54	44	50	54	38	28	33
xx55	27	18	27	26	31	18
xx56	6	7	6	6	11	7
xx61	9	10	18	9	6	17
xx62	10	11	14	12	12	10
xx63	17	19	20	19	26	22
xx67	2	3	1	2	2	0
xx71	25	41	47	54	<b>44</b>	39
xx72	4		2	3	5	5

SSC	1970	1971	1972	1973	1974	1975
xx81	6	8	15	6	8	9
xx82	0	31	48	30	33	25
xx91	21	31	26	22	12	22
xx95	72	88	57	56	63	34
1101	31	42	46	34	52	45
1102	0	1	1	1	0	2
1103	1	0	0	1	5	4
1301 1302 1304 1305 1307 1308 1306	14 0 5 3 5 1 8	23 4 2 3 4 1	23 3 6 2 3 0	13 2 3 3 3 3 8	13 4 6 3 0 1 9	13 2 5 1 3 1

#### APPENDIX C

## Alternative Derivation of Equation (14) of Paragraph V.A

The following derivation omits any subscripts. The events and probabilities refer to one SSC j of one graduation-year and are derived for one year k after graduation. The results apply to any i, j, and k observable as stated in paragraph V.A.

Let P(NLO) be the probability that an officer still serves in year k after graduation. Let P(R) be the probability that an officer reaches his retirement age between graduation and year k and actually retires for that reason.

Let

$$P(ALO) = 1 - P(NLO)$$
.

Then the total probability

$$P(R) = P(R|NLO)P(NLO) + P(R|ALO)P(ALO)$$

However P(R|ALO) = 0 because an officer who left the forces due to other-than-retirement reasons prior to reaching his retirement age can obviously not retire. Thus

$$P(R) = P(R|NLO)P(NLO)$$

Now

$$P(R|NLO) = \frac{P(RANLO)}{P(NLO)}$$

and thus

$$P(R) = \frac{P(R \cap NLO)}{P(NLO)} P(NLO)$$

From the data observed

$$P(R_0 NLO) = \frac{ALRO}{N}$$
 and

$$P(NLO) = \frac{N - ALO}{N}$$

Thus

$$P(R) = \frac{ALRO/N}{(N - ALO)/N} (1 - P(ALO))$$

In order to obtain the probability that an officer will be lost between his graduation year and the year k after graduation, denoted by P(AL) only P(ALO) has to be added, that is

$$P(AL) = P(ALO) + P(R)$$

This statement corresponds with equation (8) in paragraph V.A. Now

$$P(ALO) = \frac{ALO}{N}$$

and thus

$$P(AL) = \frac{ALO}{N} + \frac{ALRO}{(N - ALO)} (1 - \frac{ALO}{N})$$

In terms of loss rates this equation is equivalent to equation (14) of paragraph V.A.

# APPENDIX D TABLES OF LOSS RATES AND THEIR ESTIMATES

Appendix D shows in six sections (D-1 to D-6)--one section for each graduation year--loss rates and the estimates of the rates conditioned on the graduation year and conditioned on the Sub-specialty code an officer has obtained.

- Section D-1: Graduation year 1970 and k = 1, 2, ..., 10; (Tables 1 to 3)
- Section D-2: Graduation year 1971 and k = 1, 2, ..., 9;

  (Tables 1 to 3)
- Section D-3: Graduation year 1972 and k = 1, 2, ... 9; (Tables 1 to 3)
- Section D-4: Graduation year 1973 and k = 1, 2, ... 7; (Tables 1 to 3)
- Section D-5: Graduation year 1974 and k = 1, 2, ... 6; (Tables 1 to 3)
- Section D-6: Graduation year 1975 and k = 1, 2, ... 5; (Tables 1 to 3)

The content of the tables was explained in chapter V.C.1 and is explained in the respective headings of the tables within the six sections. An entry of 99.0 within the tables indicates that a rate could not be calculated because nobody having the SSC under consideration graduated in the respective graduation year.

TABLE 1 FCF GRACUATION-YEAR 70 ACCUMULATEC 1CTAL LOSSES

RON 1 : ACTUAL LCSS RATE RCW 2 : ESTIMATE

10	C.833 C.256	C.0 C.135	C.0 C.135	(.286 (.375	1.000	(.375	C.500	(-212 (-218	C.344 C.323	C.333	c.055	C.528
6	0.833	0.0	0.0	0.0	0.500	0.375	0.500	0.152 0.158	0.313	0.333	0.0	0.50C 0.528
80	0.833 0.256	0.0	0.0	0.0	0.0	0.375	0.500	0.121	0.219	0.333	0.0	0.500
1	0.833 0.256	0.0	0.0	0.0	0.0	0.375	0.500	0.091	0.125	0.0	0.0	U.500 O.528
9	0.833 C.256	0.0	C.0 C.135	0.0	0.0	0.375	0.500	0.076	0.063 C.C51	0.0	C.046	0.0
ស	0.833 0.256	0.0	0.0	0.0	0.0	0.0	0.0	0.030	0.0	0.0	0.0	0.0
4	0.500	0.0	0.0	0.0	0.0 0.086	C.C.0	0.0 0.0e6	C.015 0.012	0.007	600.0	600.0	600.0
C)	6.333 0.103	0.0	00 0.0 0.0 4/2	0.0	750.0	C. C. 49	0.057	00	00	00 00	0.0	00 00
7	0.167	0.027	0.0	0.0	0.0	C.C 0.024	0.0	00	°°°	00 00	00.0	90 90
, <b>-</b>	J0:0	0.0	J.0 0	J.0 0.0	JO:	)•°0	00.0	50.0	J.0	J.0 0	J.0 0.0	J.0 .0
SSC	21	22	23	24	25	3.6	23	<del></del>	32	(1) (1)	34	ω "1

C.222 C.202	55.000 C.207	(.500 (.526	C.163 C.181	(.500 (.305	C.500 C.403	C.318 C.322	(.370 (.398	(.167	C.333 C.441	(.300 (.252	C.529 C.462	C.0 C.275	C.400 C.375	C.250 C.455
0.133	99.000	0.500	0.047	0.50C C.284	0.357	0.318 0.36	0.370	0.167	• •	0.300	• •	• •	0.320 0.30C	0.250
0.111	99.000	0.250		0.500	0.286	0.273	0.256	0.167	0.0	0.300	0.254	0.0	0.200	0.250
0.044	99.000	0.250	0.0	0.500	0.214	0.250	0.256	0.0	0.0	0.300		0.0	0.160	0.250
C.044 0.025	95.000	0.00	• •	0.0	0.214	0.114	C.037 0.083		0.0	0.100		0.0	0.040	0.250
0.022	99.000 0.011	0.000	0.0	0.0	0.0	0.0	0.037	0.0	0.0	00.0		00		0.0
0.022	9.000	0.000		00	00	00	00	00	00	00 00		• •	90	00
C. C22 0.C15	\$5.000 0.011	0.000	0.007	00.0	0.0	00	000	00 00	0.0	00 00	ပ <b>ု</b>	0°0 0°0	٥٠ ٥٥	00 00
00	0.00	00	٥٠° ٥٥°	90	00	00	000	00	000	00	000	00	00.0	00
90 90	);°°55	00 00	00	00	90 90	00	00	00	J0 J	0 · 0 · /	00 00	00 00	JO JO	ပဂ္ဂ ပဂ္ဂ
45	7 7	8	64		(V)	4	n)	56	19	29	63	19	7.1	72

00	000.55	(.333 (.263	C.236 C.256	C-161 C-16C	951°3 C•156	• •	(.286 (.255	C.250 C.267	C.400 C.413	C.333 C.345	C.021	C.600 C.522	C.024
00	99.000	0.286	0.222	0.129 0.126	99.000	0.0	0.143	0.0	0.0	0.333	0.0	0.600	0.0
00	99.000	0.238	0,181	0.057	99.000 0.054	0.0	0.071	0.0	0.0	0.0	0.0	0.400	0.0
00	99.000	0.150	0.111	0.057	99.000	0.0	0.071	0.0	0.0	0.0	0.0	0.200	0.0
00	0.00	0.055	0.042	0.063	55.000	0.0	00	00	00	00	00 00	00.0	00
00	0.00	00	00	00	99.000	00	00	00.0	000	00.0	00	0.0	00
00	0.00	00	00	00	0.00	00	00	• •	00	) • 0 • 0	90	00	ပင
00 00	0.0	00	000	00	0.0	00	000	000	00 00	00	 	00.0	00 00
00	0.0	00	000	00	0.0	• •	00	• •	00	000	00 00	000	00
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81	8 2	5.1	<b>4</b> 1	1101	1102	1103	1301	1302	1304	1305	1306	1367	1368

TABLE 2 FCR GRADUATION-YEAR 70 ACCLMLLATED LCSSES (EXCL. FETIREMENT)

RGh 1 : ACTUAL LCSS RATE ROW 2 : ESTIPATE

01	(.833 (.256							C.045				
σ	0.833							0.045				
80	0.833 0.256	0.0	0.0	0.0	0.0	0.0	0.0	0.045 0.052	0.054	0.0	0.0	0.056
1	0.833	0.0	0.0	0.0	0.0	0.0	0.0	0.045	0.094	0.0	0.0	0.0
9	0.833	0.0	0.0	0.0	0.0	0.0	0.0	0.045	0.062	0.0	0.0	0.0
S	0.833	0.0	0.0	0.0	0.0	0.0	0.0	0.030	0.0	0.0	0.0	0.0
4	0.500	0.0	0.0	0.0	0.0	0.0	0.0	0.015	0.0	0.0	600.0	0.0
3	0.333	0.0	0.0	0.0	0.0	0.0	0.0	00	ပပ <b>• •</b>	00	000	000
2	0.167	0.0	0.0	0.0	0.0	0.0	0.0	00	00	000	00	00
-	 	0 • • •	 	0 0 0 0	ن ن ن	٥٠ ن	00 00	ပပ ပပ	ပပ ၁၁	33	ن ن ن	ပ <b>်</b>
S S C	21	22	23	54	25	26	27	31	32	m m	34	38

C.085 C.066	5 5 . 0 0 0 C . 0 5 4	C.052	C.023	(.500 (.305	C.429 C.318	(.255 (.299	(.255 C.292	C.167 C.293	(-111 (-255	(.300 C.292	(.412 (.327	(.0 C.275	(.36C (.333	C.273
0.067	99.000	0.0	0.023	0.5CC 0.284	0.286	0.285	0.255	0.167	0.111	0.366	• •	0.0	0.286	0.0
0.067	99.000	0.0	0.0	0.500		00	00	0.167	00	0.300	0.254	• •	0.200	0.0
0.044	99.000	0.0	0.0	0.500	0.214	0.227	0.222	0.00	0.0	0.300	0.176	0.0	0.160	0.0
0.044	99.000	0.0	• •	0.0	0.214	0.114	0.037		0.0	• •		• •		0.0
0.022	99.000	0.00	0.000	0.0	0.0	0.0	0.037	0.0	00	00	00	00	00	
0.022	59.000	0.0	• •	00	000		00	• •	000		• •	000		00.0
0.022	• •	0.0	• •	00	00	00	00	00	00	00	00	ပပ ပ <b>ဝ</b>	000	00
00	000-65	000	00	000	00	00	00	00	000	000	000	000	00	00
00 00	)))°\$5	00 00	ပပ ပပ	0 0 0	0°0	) ) )	ပပ ပပ	0.0	 	 	 	)• <u>;</u>	00 00	 
45	77	4 &	5 %	5.1	5.2	54	e S	96	61	62	63	6.3	11.	12

000	000035	(.238 (.158	C-1111 C-127	150°3 C.095	\$60°3 C•094	160.3	C.019	(.0 C.023	C.022	(.0 C.023	C.021	C.20C	(.0 (.024
000	0.06	0.190	0.111	260.0	99.00¢ 0.094	0.0	0.0	0.0	0.0	0.0	0.0	0.200	0.0
000	99.000	0.150	0.057	0.057	99.000	150.0	0.0	0.0	0.0	0.0	0.0	0.200	0.0
00.0	99.000	0.190	0.069	0.097	99.000	0.0	0.0	0.0	0.0	0.0	0.0	0.200	0.0
00	000.66	0.095 C.044	0.014	0.065	26.000	0.0	00	00	00	00	00	00	90
000	99.000	00	00	00	99.000	• •	• •	000		000	000	• •	
00	000.65	• •	00	00	000.65		• •	00	• •	• •	00	• •	• •
00 00	000-66	00	000	00	000-65	• •	000		000	• •	000	• •	000
000	000.66		00	00	000-65	000	00	000	0.0	000	000	000	000
00 00	25.000	00 00	00 00	00	0.0	00 00	 	00 00	) ) )	ပပ ပူပ	 	00 00	 
81	62	15	35	1101	1102	1103	1301	1302	1304	1305	1366	1361	1308

TABLE 3 FCF GPACLATICN-YEAP 70

ACCUMULATEC LCSSES

ROW 1 : ACC.LCSS RATE CF SSC-(RP INCICATED RCW 2 : ACC.LCSS RATE(OTF FR) CF SSC-GRP ROW 3 : ACC.LCSS RATE (RETIRE) OF SSC-GRP ROW 3 : ACC.LCSS ROW 3 :

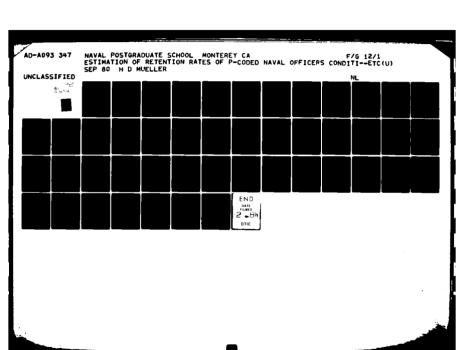
S SC	1	2	3	4	5	6	7	8	9	10
20	ς.ς 8.δ	0.030 0.030	0.061	0.031 0.031	0:152	0.273 0.143 0.143	0.273 0.152 0.143	0.273 0.143 0.143	0.302 0.152 0.175	(.394 (.152 (.286
30	6.0 6.0	G.C 0.0 0.0	g. g g. g	0.009 0.009	0.019 0.019 0.0	0.066 0.041 0.020	0.104 0.057 0.050	0.160 0.057 0.110	0.208 0.057 0.160	C.255 C.057 C.210
40	0.0 0.0	0.0 0.0	0.011		0.011	G.C22 0.022 0.0	0.033	0.076 0.033 0.045	0.109 0.042 0.068	C.207 C.054 C.161
50	6.6 6.6	0.0 0.0	0.0 0.0	0.0	0.011	0.057	0.247 0.215 0.041	0.280 0.247 0.643	0.332 0.280 0.075	6.355 6.301 6.077
6C	0.C	0.0 0.0	0.0 0.0	0.0	0.0 0.0	0.105	0.158 0.158 0.0	0.211 0.211	0.316 6.285 0.037	(.395 (.289 (.148
70	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0	.0.065 0.034 C.036	0.172 0.138 0.040	0.2C7 0.172 0.042	0.31C 0.241 0.091	C.379 C.310 C.100
\$C	6.C	0.0 0.0	C.C.	0.0	0.0	0.0 C.0	0.0	0.0 0.0	0.0	C.0 C.0
50,	6.6 6.6	0-0 0-0 0-0	0.0 0.0	C.C 0.0	0.0 0.0	C-C54 Q-Q32 Q-Q22	0.129 0.097 0.036	0.154 0.118 0.085	0.237 0.125 0.123	C.258 C.140 C.137
110C	0. C	0.0 0.0	0.0 0.0	0.0	0.0 0.0	E 90.0 E 90.0	0.054 0.094 0.0	0.054 0.094 0.0	0.12 0.094 0.034	(-156 (-094 (-069
1300	0.C 0.C	0.0 0.0	0.0 0.0	0.C 0.0 0.0	0.0 0.0 0.0	0.0 0.0	0.050 0.025 0.026	0.075 0.025 0.051	0.150 0.025 0.128	C.275 C.025 C.256
70	0.C 6.C	0.002 0.002 0.0	0.005	0.009 0.009 0.0	0.016 0.016 0.0	0.671 0.055 0.017	0.132 0.100 0.036	0.173 0.116 0.064	0.224 0.133 0.105	(.281 (.144 (.160

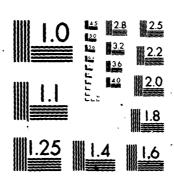
TABLE 1 FCR GRACUATICN-YEAR 71 ACCUMULATED 101AL LOSSES

ROM 1 : ACTLAL LESS RATE KGW 2 : ESTIMATE

6	0.857	0.0	~	• •	• •	000	1.000	35	0.432	433	0.667	0.333
80	0.857	0.0	0.0	0.0	.50	1.000	1.000	0.363	0.324	0.333	.33	0.0
1	0.857	0.0	0.0	0.0	-1	1.000	1.000	0.242	0.135	0.333	MU	0.0
9	0.857 C.732	0.0	0.0	50	7-	1.000	00	) I Č	0.1C8 C.C55	00	0.0	0.0
2	0.857	0.0	0-	00	0.250	000	1.000	0.136	0.081	0.0 0.058	00	0.0
4	0.429	0.0	00	0.0	0.0	ပ္ပ	ပ္ပ	ပပ္	0.0	0.0	0.0	0.0
<b>6</b> )	42	00	25 0.07		00	024 0.0	.C25 0.C	.061 0.C	C C C C C C C C C C C C C C C C C C C	0.0 710.	.c. 0.c	0.0
·	0.143 0.42	0.0 0.021 0.0	0 0.0 0.0 0	0 0.0 0.0 0	C.C 0.0 0.C23 0.C	0 6.024 0.0	0.0 625.0 0	0.063 0.0	0.0.0.0.0	0.0 0.0 0.0	0.0 713.0	.0 0.C 0.0 .0 0.C17 0.0
m	.c 0.0 0.143 0.42	0.0 0.0 0.0 0.0 0.0	0 0.0 0.0 0	.c. C.C. 0.C. 0.C. 0.C5	0.0 0.0 0.023 0.0	0.0 0.0 0.0 0.0	0.0 250.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0	0.0 0.0 0.0 0.0 0.0	0.0 E10.0 0.0 0.0 0.0	0.0 7.0 0.0 5.	0.0 7.0 0.0 0.0	0.0 7.0 0.0 0.0 0.0

J.	0.0		• •	0.029		0.087	0.145	0.174
• •		0.0	• •	0.0	0.0	0.0	0.0	0.0
		00		0.056	0-111	0.167	0.167	0.167
• •		00 00	• •	0.059	0.137	0.157	0.157	15
00		00	00 00	0.250	C-25C C-144	0.250	0.250	0.500
• •			• •	0.0	0.0	0.167	0.167	0.167
• •			• •		0.246		0.500	525
• •		000		0.0 0.084	0	0.278	0.278	0.333
00°		00	00	0.0	C-0 C-125	0.0	0.143	0.143
			00 00	0.0	0.0	0.200	0.200	0.200
		00 00		0.364	C.455 C.241	0.545	0.545	0.545
		00	00	0.105	0.211	0.316	0.421	0.474
• •		00 00	• •	0.0	0.0	0.0	0.667	0.667
• •		00	000	0.098	0.122	0.195	0.366	0.39C 0.388
		00 00	00	0.0	0.0	0.0	0.0	1.000





MICROCOPY RESOLUTION TEST CHART NATIONAL BUREAU OF STANDARDS 1963-4

0.250	19	0.355	0.318	.23	15	23	0.348	.25	0.0	.10	0.300	• •	0.0
0.0	0.129	0.250	0.216	• •	0.0	• •	0.304	00	• •	0.0	0.100	0.0	0.0 0.0 63
0.0	0.129	0.226	0.159	0.167	0.0	99.000	0.304	00	0.0	• •	0.100	• •	0.0
0.0	0.057	0-161	0.091	0.115	0.0	911.0	0.304		0.0	0.0	0-100	0.0	0.0
0.0	0.065	0.129	0.045	00	00	\$9.000 0.023	0.217	0.0	0.0	• •	0.100	0.0	0.0
• •	• •	00	00		00	0.00	• •	• •		• •	00		• •
00 00	00 00	00	00	•	00 00	0.00	ပ်ဝ	00 00	00 00		JO.		
00	00	00	• •	00	• •	0.00	00				00		
ပပ	• •	• •	00 00	• •		)))°6 )°0	• •	•			J0		
81	8 2	16	ς. 81	1101	1102		1301	1302	1304	1305	1306	1301	3061

TABLE 2 FOR CRADUATION-YEAR 71 ACCIMULATED LCSSES (EXCL. RETIREMENT)

RCW 1 : ACTUAL LOSS RATE RCh 2 : ESTIMATE

σ	0.571	0.0	0.0	0.0	0.250	0.0	0.0	0.152	0.185	0.0	0.0	$\sim$
<b>c</b> c	0.571	0-1	0.0	• •	• •	0.0	• •		• •	0-0	0~	0.0
	0.571	0.0	0.0	0.0	0.250	0.0	0.0	0.121	0.135	0.00	0.0	0.0
•	0.571	0.1	0.1	90	7	0.0	0.1	70.	70	00	90	00
w	0.571	0.0	0.0	0.00	0.250	0.0	0.0	0.061	0.081	0.0	0.0	0.0
4	0.429	0.0	0.0	0.0	0.0	0.0	0.0	0.030	0.0	0.0	0.0	0.0
6	0.143	0.0	90	00	• •	0.0	• •	• •	• •	• •	• •	• •
7	00	00	00	00	00	00	00	00	00	00	00	00
-	 	0.0	J.,	) ) )	 	3. 3.	 	0.0	90 33	0.0	00	00
u	21	22	e)	4	ξ.	9	_	<u>-</u>	2	33	4	æ

0.056	0.0	0.056	113	0.250	.16	0.381	0.222	0.0	0.100	0.545	0.368	0.0	0.244	0.0
0.058	0.0	0.056	0.137		0.167	0.440	0.222	0.0	0.100	0.545	0.316	0.0	0.244	0.0
0.029	0.011	0.056	0.137	0.250	0.167	0.340	0.222	0.0	0.100	0.545	0.263	0.0	0.122	0.016
0.014	0.0	0.056	-0	0.250	0.0	• •	0.0		0.0	0.455	0.158	0-0	0.072	0.0
0.0	0.0	610.0	0.059		0.00			0.094	0.00	0.364	0.053	0.0		0.047
00	• •	• •	•		00	• •	00	00	00 00	00	•	• •	• •	00
• •	• •	• •	• •	• •	90	• •	• •	• •	• •	• •		00	• •	• •
00	• •	00	• •	• •	00	• •	• •	• •	• •	• •	• •	• •	• •	00
• •	• •	• •	• •	• •	00 00	• •	• •	• •	• •		• •	• •	• •	• •
45	*	<b>4</b>	54	15	52	54	e S	96	19	62	63	63	11	12

0.0	.09 .08	0.226	0.136	0.167	.15	99.000	.17	50~	• •	0.0	0.0	0.0	
0.0	0.057	0.154	0.114	7.	0.0	99.000	0.174	00	0.0	0.0	0.00	0.0	.0.0
• •	0.057	0.161	0.080	0.143	0.0	99.000	0.174	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.057	0-129	0.045	0.055	0.0	99.000	0.174	0.0	• •	0.0	0.0	0.0	0.0
0.0	0.065	0.097	0.023	00	00	99.000	0.174	0.0	0.0	0.0	0.00	0.0	0.0
		• •	00	00	00	99.000	00	00	00	00	00	00	00
• •	• •	• •	• •	00	• •	99.000	• •	• •	• •		00	•	
00	00	00	00	00	00	000-65	00	00	00	00	00	00	00
 	<b></b>	90	 	99	90 90	33.58	00 00	 	 	00 00	 	);; ;;	 
81	62	15	S.	1101	1102	1103	1301	1302	1304	1305	1306	1361	1366

TABLE 3 FCF GRACUATION-YEAP 71
ACCUMULATED LCSSES

ROW 2: ACC.LCSS RATE CF. SSC-GRP INDICATED
ROW 3: ACC.LCSS RATE(OTF-FR) CF. SSC-GRP
ROW 3: ACC.LCSS RATE(OTF-

SSC	1	2	3	4	5	6	7		•
2 <b>0</b>	9.¢	0.0 0.0	9:034	0.077 6.677	0.254 0.128 0.147	0.254 0.129 2.129	0.256 0.120 8.147	0-282 0-138 0-138	0.333 0.133 0.233
30	g.g	9. G 9. G	9: 63 <del>8</del>	0-052 0-017 0-035	0-104 0-061 0-046	0-148 0-054 0-058	0.209 0.113 0.108	0-3C4 0-130 0-2C0	8:383
40	9.0 8.8	9.9 9.8	8:8	0.0 0.0	0.043 0.022 0.022	6-634 8-634	8:033	0: 1:1 8: 8:4 8: 8:1	8:08 5 8:08 5
3¢	6-6 6-3	0.0 0.0	9.C	9.9 9.9	0:112 0:101 1012	0.149 0.034	0:313 0:313	0.343 0.343 0.643	0:427 0:337 0:134
4€	9.5 9.5 9.8	g. g g. g	6.6 6.6	g.g g.g	0:110 0:116 0:026	0.255 0.185 0.255	0.324	0-419 8-367 8-167	0.442 8:325 8:172
70	9.9 8.8	9.9	0.0 0.0	0.0 8.8	0.095 0.048 0.050	0-115 8:851	0-150 8-112 8-21	0:3:7 8:238	G-4CE 8-215
•c	6.8 6.8	0-0 0-0	6.6 6.6	0-0	0:051	9:077	0-103 0-677 0-624	0-1C3 0-077 0-028	0.207 0.077 0.135
50	Ş.Ş	9.9 9.9	9.9 9.0	0.0	0-047 9-024	0.1(5 0.04)	8:131	8:133	0.32 <b>9</b> 0.160 0.200
1100	6.6 8.8	9-9 9-9	8.6. 8.6.	9.0 0.0	0.023 0.0 0.023	0-115 0-025 0-025	0:143 0:149 0:027	0.259 0.140 0.601	0.233
1300	ģ:ģ	0.0	6.6 6.6	9.9 9.0	0-128 8:047	9-17C 9:043 6:043	0-170 0-043 0-043	0-170 0-053 0-053	0.25: 0.105 0.165
71	9.g 3.5	g.g g.g	0-GC7 0-GC4 0-G03	0-013 6-067 6-064	0.091 0.059 0.034	C-137 C-072 C-049	0.157 0.131 0.076	0.256 0.158 0.16	G-313 G-113 G-144

TABLE 1 FOR GRADUATION-YEAR 72 ACCIMULATED 161AL LOSSES

ROW 1 : ACTUAL LOSS RATE ROW 2 : ESTIPATE

<b>©</b>	0.333	0.111	0.0	0.200	0.333 0.351	0.220	0.0	0.226	0.254	0.200	0.50	0.0
_	0.333	0.111	0.0	0.200	0.167	0.200	0.0	0.210	0.176	0.200	0.500	0.0
•	0.333	0.0	0.0	0.027	C.C 0.026	C.2C0 0.222	0.030	6-177	C. C88 0.100	0.200	0.566	0.073
Ŋ	0.333	0.0	0.0	0.0	0.0	0.200	0.0	0.177	0.029	0.200	0.250	0.0
*	0.333	0.0	0.0	0.027	0.00	0.0	0.0	0.129	0.029	0.229	0.250	0.0
m	0.333 0.033	0.00	0.0	0.0	0.0	0.0	0.0	0.048	0.00	0.0	0.00	0000
7	00	00	• •	00	00	00	• •	00	00	00	00	00
~	0U	• •	• •	• •	0 0 0	• •	• •	• •	• •	• •	• •	• •
SSC	21	22	23	24	52	56	2.1	31	32	33	34	38

,₫.

0.212	99.000	0.362	0.227	0.051	0.316	0.444	0.333	0.500	0.278	0.266	0.350	1.000	0.400	0.50
0.188	99.000	0.324	0.182	0.051	0.263	0.407	0.333	0.500	0.222	0.286	0.300	1.000	0.258	0.0
0.141	99.006	0.265	0.114	0.091	0.211	0.315	0.222	0.333	0.111	0.286	0.200	0.0	0.191	0.0
0.094	99.000	0.206	0.114	0.0	0.105	0.222	0.037	0.333	0.056	0.286	0.100	0.0	0.149	0.0
0.012	59.000	0.088	0.091	0.0	0.00	0.111	0.0	0.333	0.0	0.214	0.100	0.0	0.064	0.00
00 00	000-66	0U 0U	00	00	00	00	00 00	00 00	00 00	0 0 0 0	00	0 0 0 0	00	00
00	000.0	00	00	00	00	00	00	00	00	00	00	00	00	00
0 0 0 0	25.000	0 0 0	00 00	00 00	90	00	9 <b>.</b>	00	90	00	95 95	00	000	0 0 0 0
45	44	8	45	51	25	54	55	56	61	62	63	19	11	72

0.267	0.168	0.500	0.246	0.156	0.0	95.000	0.364	0.0.0	0.167	0:01	0.118	0.333	95.000
0.267	0.125	0.462	0.246	0.152	0.0	99.000	0.261	0.00	0.0	0.071	0.059	0.333	99.000
0.067	0.083	0.269	0.158	0.130	0.0	99.00C 0.128	0.217	0.000	0.0	0.01	0.0	0.333	99.000
0.0	0.083	0.154	0.140	0.065	0.0	99.000	0.174	0.0	0.0	0.0	0.0	0.0	99.000
0.0	0.042	0.0	0.088	0.022	0.0	59.000	0.130	0.0	0.0	0.0	0.0	0.0	950.0
00	00	00	00	00	00	99.000	00	00	00	00	00	00	93.00
00	00	00	00	00	00	000.65	00	00	00	00	00	00	000-66
 	9- 9-	 	90 00	 	0 0 0	))°55	9 <b>.</b>	30 30	33	 	 	00 00	25.666
18	85	15	5,5	1911	1102	1103	1301	1302	1304	1305	1306	1301	1306

TABLE 2 FCF GFACUATION-YEAR 72 ACCUMULATED LCSSES (EXCL. RETIREMENT)

RUN 1 : ACTLAL LCSS RATE ROW 2 : ESTIMATE

<b>c</b> c	0.333	• •	0.0	0.0			0.0	• •	0.059	• •	• •	• •
-	0.333	0.0	0.0	0.0	0.0	0.0	0.0	0.113	0.059	0.0	• •	0.0 0.083
٠	0.333	0.024	.02	0.0	0.0	0.0	0.0	0.057	0.059	0.0	• •	0.0
•••	0.333	• •	0.0	0.0	0.0	0.0	0.0		• •	0.0	0.0	0.0
4	0.333	• •	0.0	0.0	0.0	0.027	0.0	0.065	0.0	0.0	0.0	0.0
M	6.333	0.0	• •	0.0	0.0	0.627	0.0000	0.016	0.007	• •	• •	600.0
8	90	• •	• •	• •	00	• •	• •	<b>50</b>	• •	• •	00	. 4
-	90	• •	• •	• •	90 90	• •	• •	• •	• •	• •		• •
286	21	17	23	24	2,5	56	21	<del>~</del>	25	e)	34	36

•														
0.129	ŏŏ	.26 .18	- 15	0.091	31.	60	0.111	90	77	0.214	0.250	.0 .2¢	0.255	0.0
0.118	99.000	0.265		0.091	0.263	0.352	0.111		0.167	0.214	0.200	0.0	0.170	0.0
0.094 C.113	99.00C	0.235	0.1	0.091	0.211		0.074	• •	11	0.214	0.150	0.0 0.14E	• •	0.0 C.CSR
0.071	99.000	0.176	00	• •		0.204	00	0-	• •	0.214	0.100	0.0	90.	0.0
0.0	59.000	0.059	603	0.0	00		OÒ	0.0	00	0.214	0.100	0.0	0.021	0.00
00 00	020.85	• •	• •	• •	• •	• •	00 00	• •	• •	• •	• •	• •	00	
00	0.00	• •	• •	• •	• •	• •	90 90	• •	• •	• •	• •	00		
 	33.66	-0 -0	• •	• •	• •	• •	00° 00°	• •	• •	• •	00° 00°	• •	• •	
45	4	<b>4</b> E	54	51	(N M)	4	u: u:	75	<b>61</b>	62	63	13	11	7.2

0.0	0.083	0.308	0.105	0.174	0.0	99.000	0.174 0.164	0.000	0.0	0.0	0.0	0.0	99.000
0.0	0.083	0.269	0.105	0.130	0.0	99.000	0.174	0.0	0.00	0.0	0.0	0.0	99.000
0.0	0.083	0.231	0.053	0.109 0.108	0.0	99.000	0.174	0.0	0.0	0.0	0.0	0.0	99.000
0.00	0.083	0.115	0.053	0.065	0.0	99.000	0.130	0.0	0.00	0.0	0.0	0.0	99.000
0.026	0.042	0.000	0.018	0.022	0.0	55.000	0.087	0.0	0.0	0.036	0.0	0.0	\$5.000
J0 J0		JO.	00	00 00		000.56	000	0.0	0°0 0°0	JO 20	<b>00</b>	00 00	0.0
• •	00	90	• •	00	• •	• •	00	• •	00	000	00	00	0.0
00 00	J0 0	99 99	90 90	00	• •	0.0	00	50 50	) 0 0	0.0	<b>90</b>	90 90	)
81	85	16	45	1011	. 102	1103	1301	1302	1304	1305	1306	1367	3061

TABLE 3 FGR GRACUATION-YEAR 72

ACCUMULATED LESSES

ROB 1: ACC.LESS RATE OF SSC-GRP INDICATES

ROB 2: ACC.LESS RATE (TETRES) CF SSC-GRP

ROB 3: ACC.LESS RATE (TETRES) CF SSC-GRP

ROB 3: ACC.LESS RATE (TETRES) CF SSC-GRP

SSC	1	2	3	4	, 5	6	7	8
20	9.6 6.6	C.C G.G	0.031 0.031	0.031 0.031	0.062 0.031 0.032	0.042 0.031 0.032	0.156 0.031 0.129	6:157 8:531
30	۶. و ه. و	0.0 0.0	0-025 0-025 0-019	0.103 0.037 0.048	0.131 8.034 8.079	0.019 0.019 1.019	0.206 0.084 0.133	0.252 0.064
46	6.C 6.G	6.6 0.0	0.0 0.0	0.049 0.031 0.019	0.123 0.098 0.027	0.160 C.123 C.042	0.215 0.160 0.066	0.252 0.166 0.103
50	6-6 9-8	0.0 0.0	0.0 0.0	0.048 0.051 0.018	0.145 0.111 0.038	G.256 G.188 G.664	0.342 0.239 0.135	0.348 0.256 0.149
46	6.9 6.9	0.0 0.0	0.0 0.0	0.094 0.094 0.0	0.132 0.113 0.021	C-189 0-151 0-644	0.283 0.189 0.116	0.121 0.208 0.143
76	0.0 6.6 6.6	0.0 0.0	0.0 0.0	0.041 0.020 0.042	0.143 0.062 0.067	0.184 C.162 0.091	0.286 0.163 0.146	0.408 0.245 0.216
86	0.0 6.6	0.0 0.0	0-0 0-0	0.032 0.032	0.063 0.063 0.0	0.079 0.063 0.017	0.159 0.063 0.162	0.2C6 0.063 0.153
SC	9-8 3-8	8.8	6-6 8-8	0.060 0.012 0.049	0.145 0.072 0.078	0.193 0.108 0.095	0.313 0.157 0.166	0.325 0.169 0.160
110G	0.0 6.6	0.0 0.0	0.0 0.0	0.021 0.021	0.064 0.064 0.0	0.126 0.166 0.024	0.149 0.128 0.024	0.151 0.170 0.026
1360	G.C G.C	0.0 0.0 0.0	0.0 0.0	0.036 0.037 0.019	0.074 0.056 0.020	0-111 0-074 0-04C	0.148 0.074 0.680	G. 2C4 0.074 0.140
72	ç.ç 6.8	0.0 0.0 0.0	0.003 0.003 0.003	0.036 0.036	0.117 0.081 0.040	0.165 0.060	0.237 0:142 0:111	0.279 0.156 0.145

TABLE 1 FCF GRADUATION-YEAR 73 ACCUPULATED 1CTAL LOSSES

RATE
LCSS
ACTLAL ESTIMAT
ROR

									•			
-	99.000	0.091	0.500	00	00	00	0.500	0.136	0.208		0.333	0.500
9	95.000 0.088	0.091	C.250 C.25C	0 0 0 0	00	00	0.500	0.136	0.042	0.571 C.327	0.333 C.26E	0.500
5	99.000	00	0.250	00	00	00	0.500	0.119	0.042 0.088	0.429 0.084	0.333	0.250
4	59.000	00	G.250 G.250	00	000	00	0.500	0.068	0.042	0.286	0.0	6.250
m	650°0 000°65	00	C. 250 C. 250	00	00 00	00	0.500	C.C17 0.023	0.042	0.143	0.00	0.0
7	000.55	00	00	00	00	00	00 00	0.017	00	00	ပ <b>ု</b>	00 00
~	33.55	J0 J0	J.0 0.0	J0	J. J. J	U0 U0	• •	• •	• •	J.0	); 0	•
286	2.1	22	23	24	55	26	12	<del>-</del>	17	(T)	34	36

0.260	99.000	0.174	0.194	0.182	0.400	0.421	0.577	0.167	0.222	0.583	0.368	1.000	0.352	0.333
0.219	99.00C	0-174	0-139	0.091 C.224	0.400	0.316 C.318	0.462 C.444	0.167	0.222	0-417	0.211	1.000	0.255	0.333
0.135 0.137	99.000	0.130	0.111	0.0	0.267	0.237	0.192	0.0	0.111	0.333	0.211	0.500	0.148	0.333
0.054 0.092	65.000	0.087	0.028	0.0	0.0	0.032	0-115	0.0	0.111	0.167	0.053	0.0	0.014	0.333
	•													
C.C21 0.021	55.000 0.013	00 00	00 00	0.00	0.000	0.026	C.C.77 0.C84	0.000	 	ပဝ	 	 	C. C37 0.037	0.017
00.	\$.000 0.013		• •	200	200	.62	283	0.0	• •	• •	• •	• •	63	0
.0 C.C2	9.000 95.000 0.0	00	90	00.00	0000	.0 0.62	0.0 0.0	0.0	0000	00	00	0000	0.0	0.0

	333	0.273	0.357	900	00	∞	0.077	O G	0.0	٧.	-12 -16	0.0	93
0.167	C.3CC 0.281	C.273 0.28C	C.286	C.088 0.086		C.0 0.081	P-9	C.5CC 0.081	6.0 0.0 0.0	• •	0.047	0.0	0.0
0.0	0.233	0.136	0.250	0.088 0.086	• •	0.0	0.077	0.500	• •	0.0	0.0	0.0	• •
0.048	0.133	0.045	0.071	0.059	• •	0.0	000	• •				00	
<b>00</b>	0.067	C.C45 0.020	0.00	0.029	0.627	0.027	• •	• •		• •	• •	00 00	• •
• •	• •	00	• •	• •	• •	• •	• •	• •	• •	• •	• •	00	• •
• •	• •	• •	• •	• •		• •	• •	• •	• •		• •	00 00	
8 1	82	5.1	ي م	1101	1102	1103	1301	1302	1304	1305	1306	1301	1306

TABLE 2 FGR GRACUATION-YEAR 73 ACCLMULATED LCSSES (EXCL. RETIREMENT)

RGW 1 : ACTUAL LCSS RATE ROW 2 : ESTIMATE

7	99.000	00	00	00	00	00	00	0.051	0.042	0.429	0.167	0.250
•	0.066	00	00	00	00	00	00	0.05	0.0	0.425	0.167	C.25C
Ŋ	99.000	00	00	000	00	00	00	0.034	0.0	0.429	0.167	0.0
4	0.00	00	00	00	00	00	00	0.017	0.0	0.286	0.0	0.0
m	000-66	00	00 00	00	00	00	00	0.0	0.0	0.143	0.0	0.0
8	000*65	00	00	00	00	00	• •	00	000	00	00	00
***	) ) ) ) ( )	 	00 00		90 90	 	 	ن ن ن	33	 	30	00 00
SSC	21	22	23	54	52	56	12	31	32	n)	34	38

0.146	99.000	0.087	0.111	0.182	0.400	0.342	0.385 0.336	0.0	0.0	0.500	0.211	0.0	0.255	0.0
0.125	99.000	0.087	0.083	0.091	.46	0.237	0.269	0.0	0.0	6.333	0.158	0.0	0.185	0.0
0.063	99.000	0.043	0.083	0.0	0.267	0.184	0.038	0.0	0.0	0.250	0.158	0.0	0.093	0.0
0.042	59.000	0.043	0.028	0.0	0.0	0.079	0.038	0.0	0.0	0.083	0.053	0.0	0.037	0.0
0 0 0 0	99.000	• •	0U 0U	0.00	0.0	0.026	0.0	0.00	00	0U 0U	• •	0 0 0 0	0.019	0.0
00	99.000	00	• •	00	• •	• •	• •	• •	• •	•	00	• •	00	• •
 	25.65	ပပ ပပ	90 90	 	• •	• •	• •	• •		• •	• •	 	 	
~	_	w		-	~	•	41	y		i V	_	_	_	~

Ď	0.267	• •	0.156	0.088 0.088	• •	00	0.077	0.500	• •	0.0	0.0	0.0 0.053	0.0
.16		0-182		0.088	00	00	0.077	080	0.0	00	0.00	0.0	0.0
0.0	0.167	13	0.179	0.088	0.0	0.0	0.077	0.500	0.0	0.0	0.0	0.0	0.0
00	0.067	0.045	0.054	0.059	• •	• •	00	00	• •	• •	00	• •	00
• •	00 00		0.00	0.029	• •	• •	• •	• •	• •		00 00	• •	ပပ ပပ
• •	90	• •	00	• •	• •	• •	00	• •		• •	• •	0 0 0 0	00
• •	• •	• •	 	• •	• •	• •	• •	• •	• •	• •	• •	J.,	J0
£ 1	, <u>, ,</u>	15	»;	1311	1102	1103	1361	1305	1364	1305	1366	1361	1361

TA	OLE	3	FCR	-	I TAJE	CK-YE	48	73		
AC	CUPI	LA	7 EC	LCSS	ES					
AG	2 1	: 4	ÇÇ.	FESS	RAIS	. ÇF. §	şç-	iar I	MOLCAT	ŧ
ÄŠ	5,5		ÇÇ.	133	ŖĢţĒ	Ţġţ,	ige	[5_0]	NOTCAT C-GRP SSC-G AR GRO	2

55C	1	2	2	4	5	•	7
26	ۇ <u>:</u> ۇ	g. g	G. (59 0.6 0.659	9.959 8.9 8.059	0.059 0.0 0.059	9.088 6.088	0-118 0-118
30	9.9 9.4	0.010 0.010	8-616 8-616 8-616	0-69C 0-030	0-140 0-060 0-065	8:075	0-225 0-040 0-143
46	8:8	9.9 8.8	8:813 8:813	0-017 8:637 8:648	0-129 8-062 8-064	8:119	8:133
90	9.6 9.6	6.6 6.8	8:010 8:010	0.C42 9:042 8:022	0: 157 8:371	8:125	8:133 8:133
40	9-C 6-6	0.0 0.0	0 . g	0.048 0.048 0.050	8:239 8:[43 8:111	8:315	0.429 0.230 0.230
76	6-6 6-8	8:8	\$13:3 \$10:0	8:533	8:37	8:113	8:331 8:148
•€	9:5	6:0 6:0	6.63e	8:833	8:133		8:355
16	ç. 0 6. 8	9-9 8-8	0.013 8:8 <sup>13</sup>	0.044 8:814	8: 32 2	8:121 8:121	8: [33
FIEC	9:9	G.G G.G	8:63 <b>8</b>	8:832	0.003	8:983	8:883
1300	9-9 6-9	0.0	0.0 0.0	0.0 0.0	0-057 0-057 0-0	6:037 6:037	0.114 0.057 0.041
73	9:5 8:8	0.001 6.001	8:83; 8:83;	0.072 8:637 8:638	0-152 0-053 0-066	8:94 8:134 8:550	8:273 8:124

TABLE 1 FOR GRADUATION-YEAR 74 ACCLMULATED 1CTAL LCSSES

HCh 1 : ACTLAL LCSS RATE ROW 2 : LSTIPATE

•	00		00	90	1.000	0.0	00.	:13	27	.50	m.	5.57
ın	0.0	04	0.0	0.0	1.000	.02	.02	14	0.182	.50	404	0.429
•	0.0	100	0.0		0.0	• •	.02	.05	00	50	.33	42
m	• •	00	00 00	• •	0°0	• •	• •	95		٠ <u>٠</u>	0.0	-28
8	00	• •	00	• •	00	• •		• •		00	0.00	00
-		• •	30	• •	• •	• •	• •	• •	• •	• •	95	• (
S S C	21	22	23	54	52	56	12	12	32	33	34	38

0.222	99.000	0-196	0.107	0.333	0.276	0.425	0.415	0.091	0.0	0.333	0-423	0.500	0.341	0.200
0.167	99.000	0-120	0.036	0.222	0.056	0.286	0.290	0.091	0.0	0.250	0.269	0.500	0.273	0.200
0.074	59.000	0.00	0.036	0.111	0.0	0.107	0.065	0.0	0.0	0.0	0.077	0.0	0.136 C.130	0.0
00.	94.000	<b>0</b> 0	90	0.0	0.0	0.036	0.065	0.0	00 00	00	0.038	00	C.068	0.0
00	000.65	00	00	0.00	0.0	0.00	0.032	500-0	00	00	0.038	00	0.045	0.0
 	33.55	<u>ن</u>	0U 0U	 	90	<b></b>	• •	33	90	00 00 00	 	 	• •	 
42	4	46	6 4	7	25	4.2	55	26	61	62	63	19	11	12

22		0.38	0.27	0.23	99.000	.20	0.07	.50	000	0.0	00		1.000
0.0	0.121	0.083	0.175	0.173	99.000	0.200		0.250	0.0	0.0	0.0	99.000	1.000
00	00.00	0.083	0.111	0.115	59-000	0.0	0-0	0.0	0.0	0.0	0.0	\$5.000 0.028	ąυ
00 00	0.030	0.623	0.063	6.038	99.000	0.C 0.C32	00 00	00.	0U 0U	00	00	000.55	00
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<b>6</b>	6.2	15	R,	1011	1102	1163	1301	1302	1304	1305	1366	1361	1366

JABLE 2 FCR GRADUATICN-YEAR 74 ACCLMULATED LCSSES (EXCL. RETIREMENT)

ROW 1 : ACTUAL LOSS RATE RCW 2 : ESTIPATE

v			0.0	0.0 0.018	00	• •	00		0~	5	0	7.
S	0.0	0.100	0.0	00	00	05	00		0~	• 5	0-	
4	0.0	0.100	0.023	0.018	90	02	05	.03 .04	.03	20	0.0	.14
m	• •	• •	• •	• •	• •	00	• •	• •		20	20.	14
~	00	• •	• •	00	• •	00		• •	00	0.000	20.	90
-	• •	• •	• •	• •	• •	90	• •	• •		• •	• •	• •
<b>3</b> 8¢	2.1	22	23	54	87	26	7.2	31	32	33	4	38

0.185	59.00C	C-120 0-136	0.071	0.333	0.222	0.250	0.250	0.0	0.0	0.333	0.231	0.0	0.205	0.0
0.148	99.000	0.080	0.036	0.222	0.056	0.143	0.194	0.0	0.0	0.250	0.115	0.0	0.182	0.0
0.056	0.0000	0.040	0.036	0.038	0.0	0.0	0.065	0.0	0.0	0.017	0.038	0.0	C-114 0-108	0.0
90 90	000.0	00 00	00	0.0	0.017	0.016	0.031	0.0	00	- - - - - - -	00	00 00	0.045	0.0
90 90	000.65	00	00	0.000	0.00	0.0	• •	0.0	00	• •	00	00	0.045	0.0
00 00	);;; ));;55	00 00	00 00	90 90	 	00 00	00 00	J.0 O.0	00 00	ပပ ပပ	 	00 00	00	ပပ ပပ
45	4	<b>4</b>	54	15	25	4	R) R)	56	6 1	62	63	61	11	12

0.08	90.0	0.116	0.00	C-23 C-22	95.00	0.20	C-C7 0-10	0.150	900	0.00	0.0	25.00	1.00
0.0	0.061	0.0	0.063	0.173	99.00C	0.200	0.0	0.250	0.0	0.0	0.0	99.000	1.000
000	00	0.0	0.063	0.115	501.0	0.00	0.00	0.0	0.0	0.0	0.0	\$5.000	1.000
00	00	0.0	0.032	0.038	99.000	0.0	00	00	00	00	00	000.66	00
00	00	0.0	0.016	0.019	99.000	0.0	00	00	00	00	00	000.65	00
9U 00	00 00	ပပ (၁ပ	0 0 0 0	00 00	ງ•ງ ງງ0•56	00 00	00	90 90	ပပ ပပ	00 00	9 9 9 9	)*) )0°55	00 00
81	82	51	55	1101	1102	1103	1301	1302	1304	1365	1306	1301	1366

TABLE 3 FOR GRACUATION-YEAR 74 ACCUMULATED LCSSES 2 2 C 2 20 0.0 0.0 0.025 0.075 C.16C 0.025 0.025 0.025 0.051 0.077 30 0.010 0.030 0.089 0.186 0.248 0.010 0.020 0.050 0.109 0.109 0.0 0.010 0.042 0.089 0.154 0.0 0.0 8:8 0.0 0.0 0.056 0.121 0.178 0.047 0.103 0.14C 0.010 0.321 0.043 50 0.010 0.031 0.062 0.216 0.351 0.010 0.021 0.031 0.134 0.237 0.0 0.011 0.032 0.095 0.149 0.022 0.022 0.043 0.239 0.340 0.0 0.0 0.022 0.130 0.217 0.022 0.022 0.022 0.125 0.167 76 80 90 0.040 0.053 0.107 0.160 0.293 0.013 0.027 0.053 0.053 0.107 0.027 0.027 0.056 0.113 0.209 1100 0.018 0.035 0.105 0.175 0.228 130C 0.0 0.0 0.028 0.054 0.111 0.028 0.056 0.111 0.0 0.0 C.C15 C.C26 Q.C71 Q.166 Q.245 Q.009 Q.015 Q.048 Q.105 Q.149 Q.066 Q.011 Q.024 Q.069 Q.112

FCR GRACLATICN-YEAR

		S	59.000 0.308	0.0	0.0	0.571	99.000	0.0	99.000	0.181 0.181	0.120	1.000	0.250	0.182
2		4	\$3.000 0.154	0.0	0.071	C.2E6 0.250	99.000	0.0	95.600	0.057	060-0	1.000	C-250 0-288	0.051
N-YEAR	ATE	m	25.000	0.059	0.671	0.143	263.68	0.0	22.000	0.042	0.029	0.034	C.C. C.025	0.024
RACLATIC Tal Loss	LCSS RA	~	0.00	00	00	00	0.00	00	000.0	0.042	C.040 0.029	0.500	0.025	0.024
FCR GR ATEC TCT	ACTUAL EST1PA	-	) )) ) ) )	JO	<b>00</b>	J.0 J.0	33.55	<b>00</b>	220.55	C.C14 0.014	JO:0	90 90	 	00 00
TABLE 1 ACCLMUL	KON 1: RGN 2:	386	2.1	22	23	24	23	26	2.1	31	(V (F)	65	34	36

0.228 0.193	0.0	0.100	0.083	0.083	0.118	0.303	0.222	0.143	0.235	0.200	0.227	99.000	0.077	0.200
0.123	0.0	0.067	0.0	0.083	0.059	0-182	0.111	0.143	0.176	0.200	0.056	\$5.000	0.077	0.200
0.0200	0.020	0.033	0.019	0.00	0.010	0.121	0.056	0.011	C.(55 0.(67	0.051	0.042	0.061	0.037	0.641
0.028	0.007	0.00	0.00	0.00	0.000	0.061	0.056	0.0	0.059	0.0	0.045	55.000	0.0	0.200
C.C18 C.C11	0.00	9000	0.0.0	90	90 90	90 90	J0 J0	00 00	00 00	90 90	00°0	)))°55	99 90	00 00
45	4	48	64	51	(2)	4	יט יע	4) 4)	61	62	63	67	11	12

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TABLE 2 FCP GRADUATION-YEAR 75 ACCLMULATED LESSES (EXCL. RETIREMENT)

RCh 1 : ACTUAL LCSS RATE RGW 2 : ESTIMATE

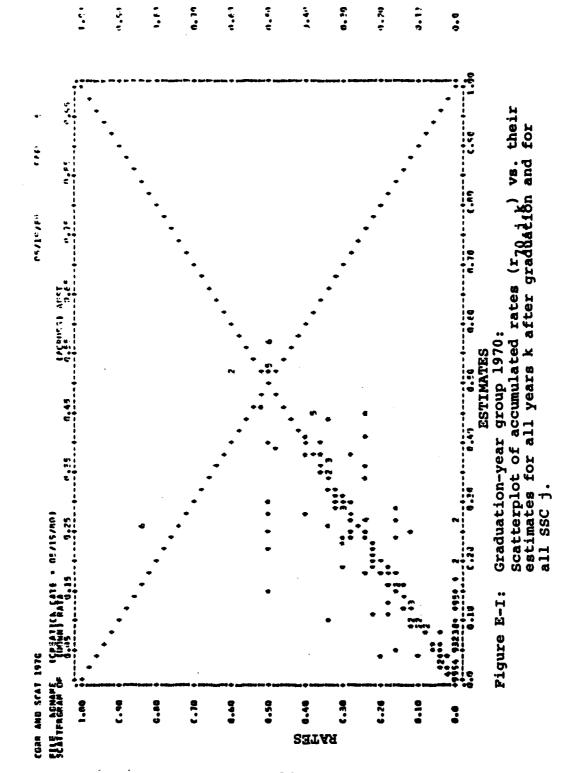
Ś	99.000	0.0	0.0	0.286	99.000 0.154	0.0	99.000	0.069	0.040	0.500	0.0	0.091
*	\$9.000	0.0	0.0	0-143	59.000	0.0	69.000	0.042	0.040	0.500	0.0	0.091
m	99.000	0.0	0.0	C.143	99.000	0.01	99.000	0.014	C. C29	C.500	0.0	0.0
7	000-66	00	00	00	000-65	00	000-65	0.014	0.040	C.500 0.034	0.0	0.0
	)))°\$\$	0 0 0 0	 	 	22.55	ပပ ပပ	) • 9 ) • 9 • 5 5	0 0 0 0	 	ن ن ن	 	ပ <b>်</b>
SSC	21	22	E 23	54	25	56	23	31	25	23	76	38

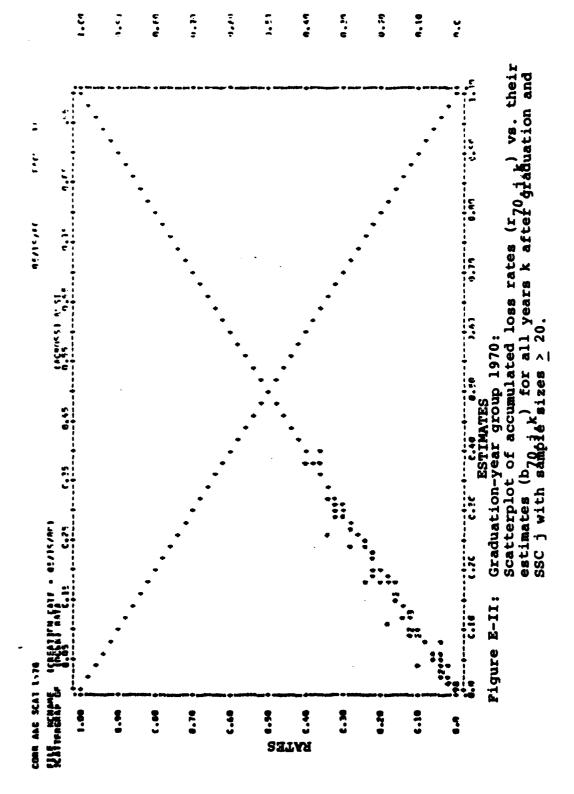
45	7 7	48	4.5	7	55	ሊ 4	R)	56	<b>61</b>	62	<b>£</b> 3	19	11	12
C.C18 0.C118	ניינים	9000	30000	ن ن ن	 	ن ن ن	ن د د	 	 	ن د ن ن	 	22255	 	UU UO
0.018	0.0	0.0	0.000	0.0	0.000	0.030 C.C17	0.00	0.0	0.0	0.0	0.045 C.028	\$9.000 0.020	0.0	0.200
0.053	0.0	0.0	0.0	0.0	0.0	C.030 0.017	0.0	0.0		0.100	0.045 C.C42	55.C00 C.C41	C.0.5	_
C. C48	0.0	0.033	03	0.083		0.061		0.0	0.0	0.200	0.045	29.000	0.051	
0.140	.07		00	0.08	0.11	0.12	0.05	0.0	0.0		0.18	99.00	0.05	0.20

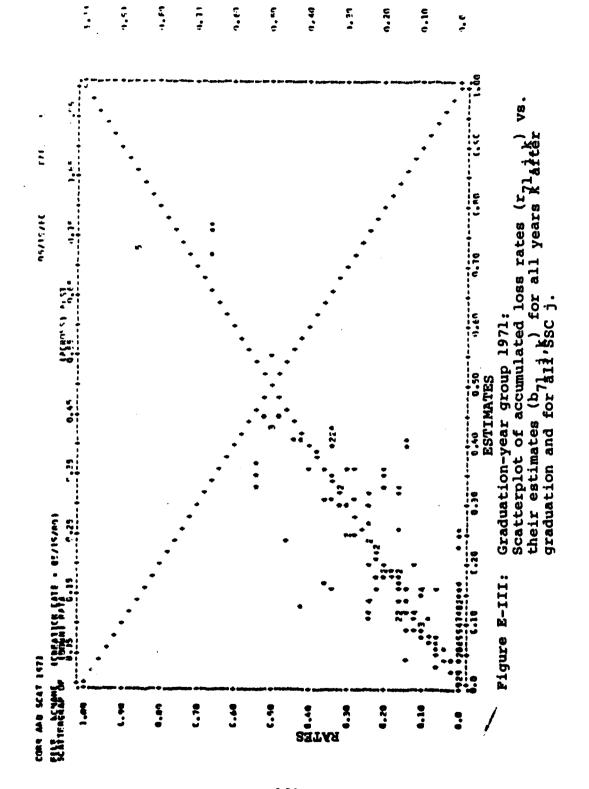
• •	• •	• •		• •	0.50	• •	• •	• •	• •	00	0.07	00	0.0
0.0	• •	00	0.029	00	• •	00	00	0.0	000	0.0	C.C71 0.038		0.0
22.		• •	00 00	• •			<b>UU</b>	22	υÚ	200	• •	200	6.C 0.C25
00 00	• •	90			00 00	• •	00	• •		0.0	• •	02	0.025
 	• •	00 00	 0	-: -: -:	ن ن ن	00	J. J	00	J0:J	J0 J0	J.J.		 
<b>4</b>	<b>62</b>	15	5.5	1101	1162	1103	1301	1362	1364	1305	1366	1361	1368

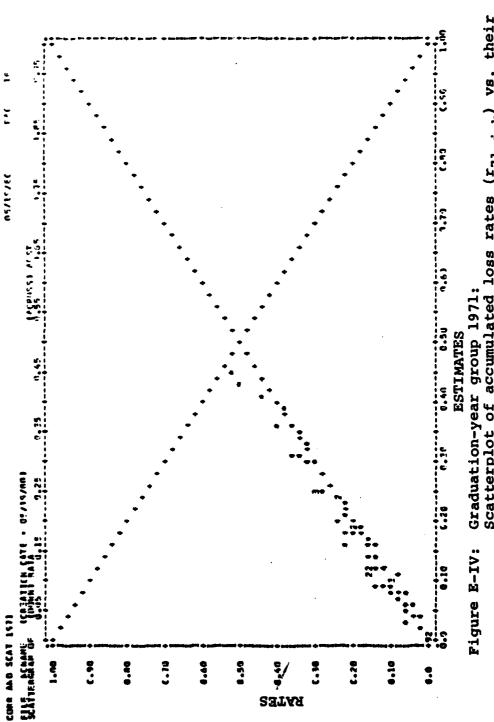
TABLE 3	FCR GF	<b>JCUATIC</b>	M-7 EAR	75	
ACCUMUL		SES			
RON 1 : RCh 2 : ROD 3 : LAST 3	ACC.LCS	S RATE S RATES S RATE ER TO T		GRP INC CF SSC- CF SSC- AG-YEAR	ICATED GRP SC-GRP GROUP
SSC		2	3	•	5
20	6. ç	6.0 6.0	G.C77 G.C77	0.134 0.077 0.063	0.300 0.150 0.162
30	932.9 93.3	0.026 0.026 0.018	0.026 0.026 0.018	0.053 0.065	0.184 0.070 0.123
40	<b>{:</b> {{ <b>!</b> }}	0.015	0.638 0.623 0.614	0.069 0.032 0.032	0.137 0.054 0.056
56	3.8 3.8	0.034 8:011	8:617	8:833	0-207 8:092 8:127
<b>ec</b>	ફ <u>ે-</u> ફ	0-041 0-020 0-021	0.641 0.641 153.3	0.122 0.061 C.C65	8:[32 8:[32 6:116
76	g. g	0.023 0.023	0.045 0.023	0.091 6.648 0.024	0.091 0.068 0.024
80	§:§	0:0 0:0	8:632	6:633 6:033 8:033	0.047
<b>90</b> .	6:5 6:6	0.0	Q.C10 Q.Ç. Q.Z.B	0.034	0.179 0.143 0.042
1100	6.6 6.6	0.0 0.0	0.c 0.c	0.078 0.039 0.021	0.137
1300	0.0 0.0	C.G26 G.G26 Q.Q	0.026 0.026 0.026	0.0 0.026 0.026	0.051 0.051
75	0.003 6.003 6.003	0.023 0.013 0.010	0+0.0 153.0 053.0	0.C27 0.047 0.042	0.162 0.091 0.078

APPENDIX E SCATTERPLOTS OF LOSS RATES VERSUS THEIR ESTIMATES









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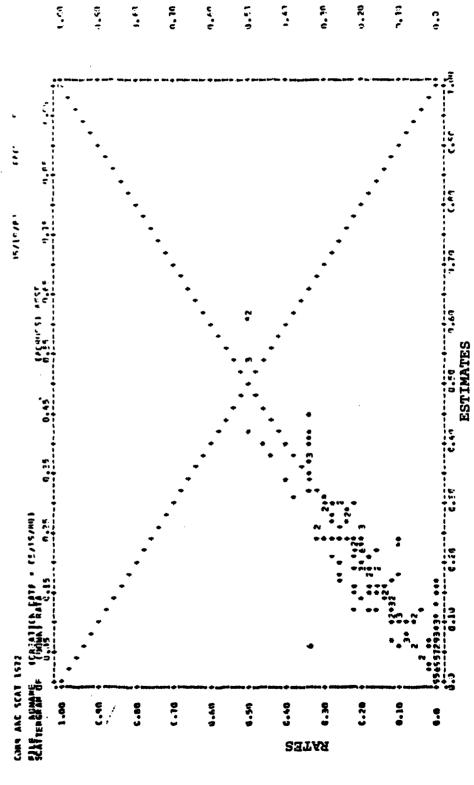
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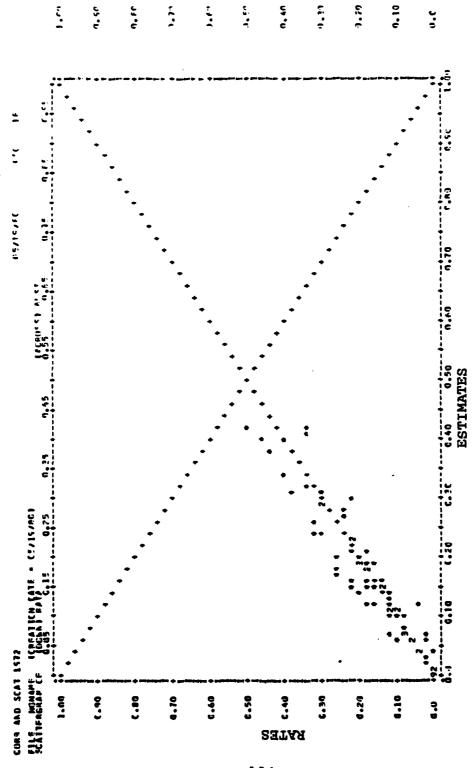
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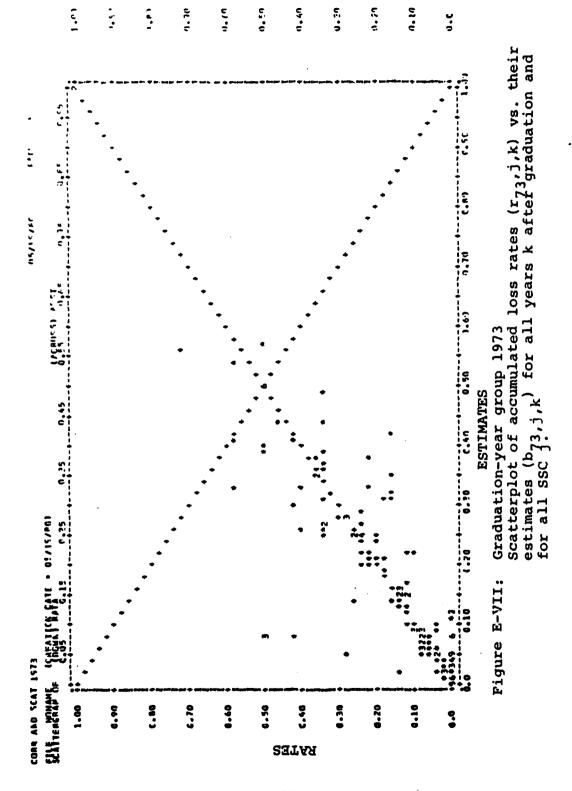
Graduation-year group 1971: Scatterplot of accumulated loss rates  $(r_{71}, k)$  vs. their estimates  $(b_{71}, k)$  for all years k after graduation and SSC j with sample sizes  $\geq 20$ .

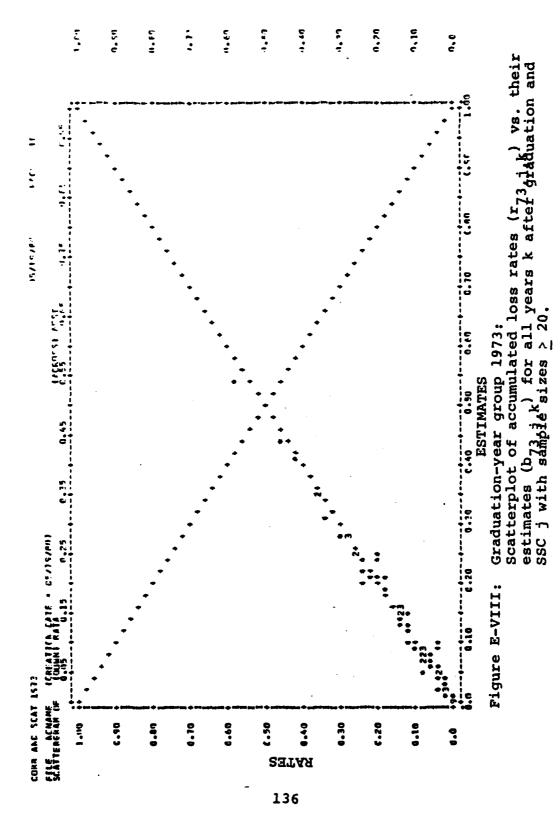


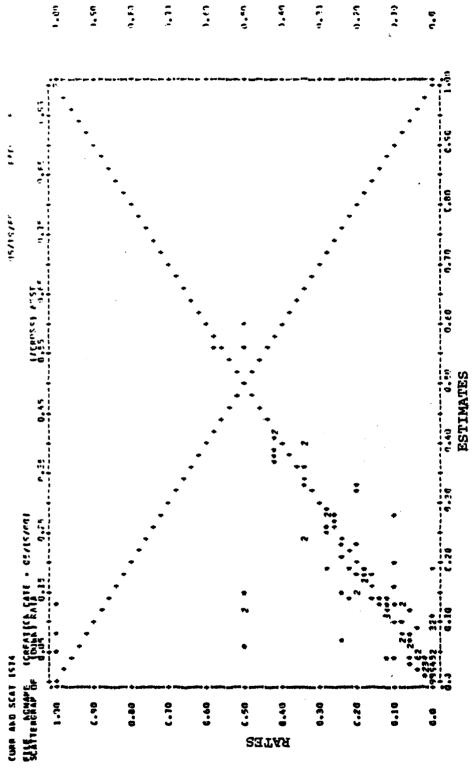
Graduation-year group 1972: Scatterplot of accumulated loss rates  $(r_{12}, k)$  vs. their estimates  $(b_{12}, j, k)$  for all years k after graduation and for all SSC  $\frac{1}{2}$ , j,kFigure E-V:



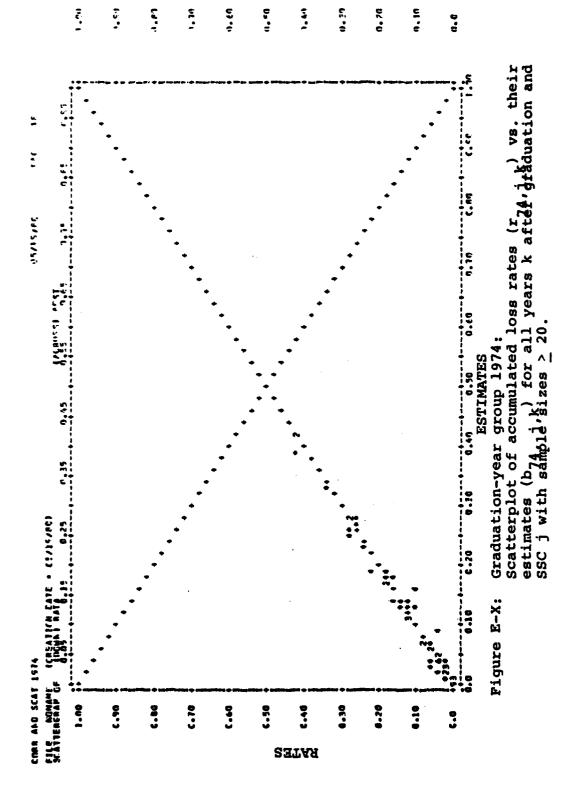
Graduation-year group 1972: Scatterplot of accumulated loss rates  $(r_{72}, \frac{k}{k})$  vs. their estimates  $(b_{72}, \frac{k}{k})$  for all years k after graduation and SSC j with sample sizes  $\geq 20$ . Figure E-VI:

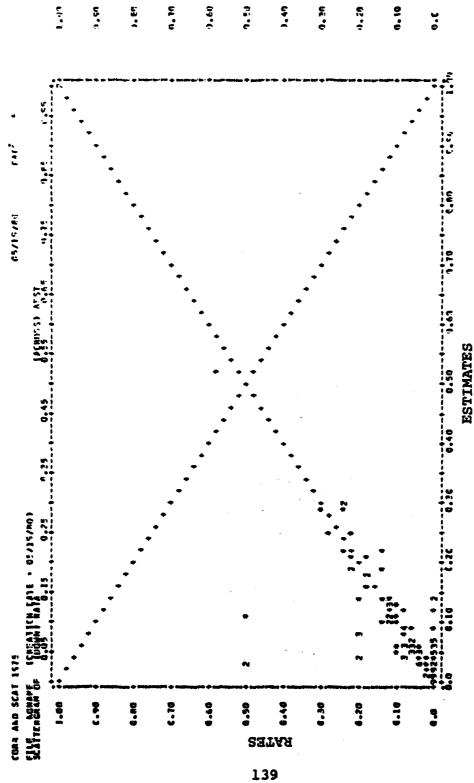




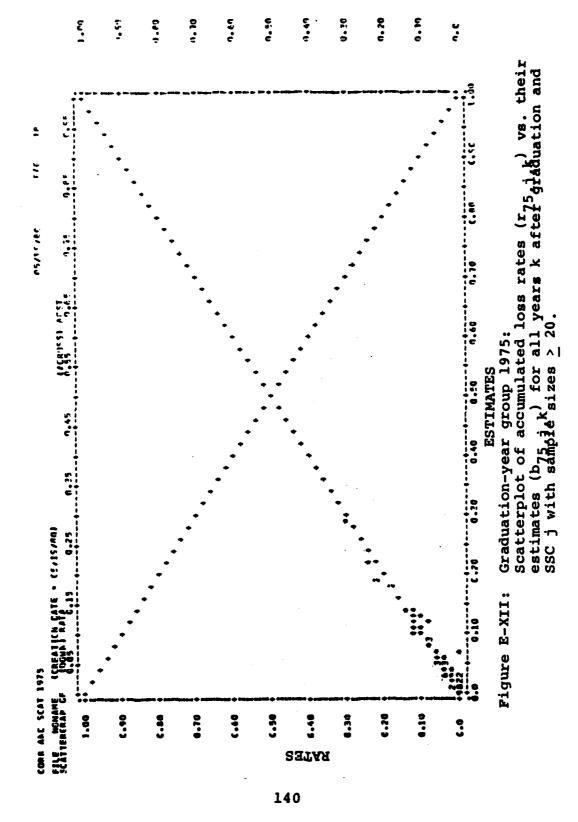


Graduation-year group 1974: Scatterplot of accumulated loss rates  $(r_{74}, k)$  vs. their estimates  $(b_{74}, j, k)$  for all years k after graduation and for all SSC  $\{j, j, k\}$ Figure E-IX:





Graduation-year group 1975: Scatterplot of accumulated loss rates  $(r_{15}, \frac{1}{12})$  vs. their estimates  $(b_{15}, j, k)$  for all years k after graduation and for all SSC  $\frac{1}{2}$ ,  $\frac{1}{2}$ ,  $\frac{1}{2}$ Figure E-XI:



## **BIBLIOGRAPHY**

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5.	CAPT Heinz D. Mueller, GAF 511 Dry Creek Road Monterey California 93940		1

## DATE ILME